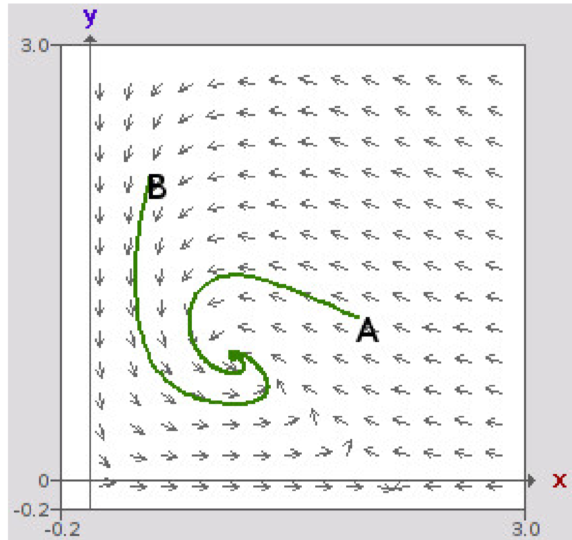
Review Questions, Exam 2

1. A weight of 4 lbs is attached to the end of a spring that is then stretched 3 inches. The spring is then attached to a dashpot that provides 6 lbs of resistance for each ft/s of velocity. If the spring is pushed up 6 inches and released, give the second order linear differential equation that models the position of the mass. (If needed, g is 32 ft/s^2 in US Customary units)

2. Consider the system below, with two solution curves shown in the phase plane.

$$\frac{dx}{dt} = x\left(2 - x - \frac{6}{5}y\right)$$
$$\frac{dy}{dt} = y\left(-1 + x\right)$$



   (a) Find the equilibrium solutions.

   (b) Given the phase plane from HPGSystemSolver with starting points A and B as shown above, sketch the solution in the (t, x) and (t, y) planes.

3. Consider the system of differential equations: $\mathbf{Y}' = \begin{bmatrix} 0 & 4 \\ -1 & 0 \end{bmatrix} \mathbf{Y}$

   (a) Find the eigenvalues to this system.

   (b) Find the general solution of this system.

   (c) Find the solution that satisfies $\mathbf{Y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

   (d) Sketch the phase plane for this system.

4. Consider the family of systems of differential equations given by:

$$(a) \quad \mathbf{Y}' = \begin{bmatrix} a & 1 \\ -1 & a \end{bmatrix} \mathbf{Y} \qquad\qquad (b) \quad \mathbf{Y}' = \begin{bmatrix} a & 1 \\ a & a \end{bmatrix} \mathbf{Y}$$

   Use the Poincare classfication to determine how the equilibrium changes. Begin by drawing the trace-determinant plane, then track where you are on the graph.

5. Given the mass-spring system with unit mass ($m = 1$), spring constant $k > 0$ and damping $b \geq 0$, write the second order linear homogeneous differential equation describing the motion of the mass.

   (a) Convert the differential equation into a system of first order equations in the form $\mathbf{Y}' = A\mathbf{Y}$ (that is, find A).

   (b) Continuing with the previous question, find the eigenvalues of A.

(c) Continuing, state all the values of b, k so that the system has complex eigenvalues.

(d) State all the values of b, k for which the system has a saddle at the origin.

6. Below is a coupled pair of second order equations in x, y. Convert it to a linear system of four first order equations. You might let u_1, u_2, u_3, u_4 be your variables.

$$\begin{aligned} x'' + 2x' - y' + x + 4y &= 0 \\ y'' + y' + 8y - 3x &= 0 \end{aligned}$$

7. Solve: $\mathbf{Y}' = A\mathbf{Y}, \mathbf{Y}(0)$ shown.

(a) $A = \begin{bmatrix} -4 & 1 \\ 2 & -3 \end{bmatrix}$, $\mathbf{Y}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Sketch the phase plane for this one.

(b) $A = \begin{bmatrix} -4 & 4 \\ -1 & 0 \end{bmatrix}$, $\mathbf{Y}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.