

## Sample Questions (Chapter 3, Math 244)

1. True or False?

- (a) The characteristic equation for  $y'' + y' + y = 1$  is  $r^2 + r + 1 = 1$
- (b) The characteristic equation for  $y'' + xy' + e^x y = 0$  is  $r^2 + xr + e^x = 0$
- (c) The function  $y = 0$  is always a solution to a second order linear homogeneous differential equation.
- (d) In using the Method of Undetermined Coefficients, the ansatz  $y_p = (Ax^2 + Bx + C)(D \sin(x) + E \cos(x))$  is equivalent to

$$y_p = (Ax^2 + Bx + C) \sin(x) + (Dx^2 + Ex + F) \cos(x)$$

(e) Consider the function:

$$y(t) = \cos(t) - \sin(t)$$

Then amplitude is 1, the period is 1 and the phase shift is 0.

2. Find values of  $a$  for which **any** solution to:

$$y'' + 10y' + ay = 0$$

will tend to zero (that is,  $\lim_{t \rightarrow 0} y(t) = 0$ ).

3.
  - Compute the Wronskian between  $f(x) = \cos(x)$  and  $g(x) = 1$ .
  - Can these be two solutions to a second order linear homogeneous differential equation? Be specific. (Hint: Abel's Theorem)
4. Construct the operator associated with the differential equation:  $y' = y^2 - 4$ . Is the operator linear? Show that your answer is true by using the definition of a linear operator.
5. Solve  $u'' + u = 3t + 4$ ,  $u(0) = 0$ ,  $u'(0) = 0$ .
6. Solve:  $u'' + \omega_0^2 u = F_0 \cos(\omega t)$ ,  $u(0) = 0$   $u'(0) = 0$  if  $\omega \neq \omega_0$  using the Method of Undetermined Coefficients.
7. Compute the solution to:  $u'' + \omega_0^2 u = F_0 \cos(\omega_0 t)$   $u(0) = 0$   $u'(0) = 0$  two ways:
- Start over, with Method of Undetermined Coefficients
  - Take the limit of  $u(t)$  from Question 6 as  $\omega \rightarrow \omega_0$ .
8. Given that  $y_1 = \frac{1}{t}$  solves the differential equation:

$$t^2 y'' - 2y = 0$$

Find a fundamental set of solutions using Abel's Theorem.

9. Suppose a mass of 0.01 kg is suspended from a spring, and the damping factor is  $\gamma = 0.05$ . If there is no external forcing, then what would the spring constant have to be in order for the system to *critically damped*? *underdamped*?
10. Give the full solution, using any method(s). If there is an initial condition, solve the initial value problem.

(a)  $y'' + 4y' + 4y = e^{-2t}$

(b)  $y'' - 2y' + y = te^t + 4$ ,  $y(0) = 1$ ,  $y'(0) = 1$ .

(c)  $y'' + 4y = 3\sin(2t)$ ,  $y(0) = 2$ ,  $y'(0) = -1$ .

(d)  $4y'' - 4y' + y = 16e^{t/2}$

(e)  $y'' + 9y = \sum_{m=1}^N b_m \cos(m\pi t)$

11. Rewrite the expression in the form  $a + ib$ : (i)  $2^{i-1}$  (ii)  $e^{(3-2i)t}$  (iii)  $e^{i\pi}$
12. Write  $a + ib$  in polar form: (i)  $-1 - \sqrt{3}i$  (ii)  $3i$  (iii)  $-4$  (iv)  $\sqrt{3} - i$
13. Find a second order linear differential equation with constant coefficients whose general solution is given by:

$$y(t) = C_1 + C_2 e^{-t} + \frac{1}{2}t^2 - t$$

14. Determine the longest interval for which the IVP is certain to have a unique solution (Do not solve the IVP):

$$t(t-4)y'' + 3ty' + 4y = 2 \quad y(3) = 0 \quad y'(3) = -1$$

15. Let  $L(y) = ay'' + by' + cy$  for some value(s) of  $a, b, c$ .

If  $L(3e^{2t}) = -9e^{2t}$  and  $L(t^2 + 3t) = 5t^2 + 3t - 16$ , what is the particular solution to:

$$L(y) = -10t^2 - 6t + 32 + e^{2t}$$

16. Compute the Wronskian of two solutions of the given DE without solving it:

$$x^2 y'' + xy' + (x^2 - \alpha^2)y = 0$$

17. If  $y'' - y' - 6y = 0$ , with  $y(0) = 1$  and  $y'(0) = \alpha$ , determine the value(s) of  $\alpha$  so that the solution tends to zero as  $t \rightarrow \infty$ .
18. A mass of 0.5 kg stretches a spring an additional 0.05 meters to get to equilibrium.  
 (i) Find the spring constant. (ii) Does a stiff spring have a large spring constant or a small spring constant (explain).

19. A mass of  $\frac{1}{2}$  kg is attached to a spring with spring constant 2 (kg/sec<sup>2</sup>). The spring is pulled down an additional 1 meter then released. Find the equation of motion if the damping constant is  $c = 2$  as well:

20. Match the solution curve to its IVP (There is one DE with no graph, and one graph with no DE- You should not try to completely solve each DE).

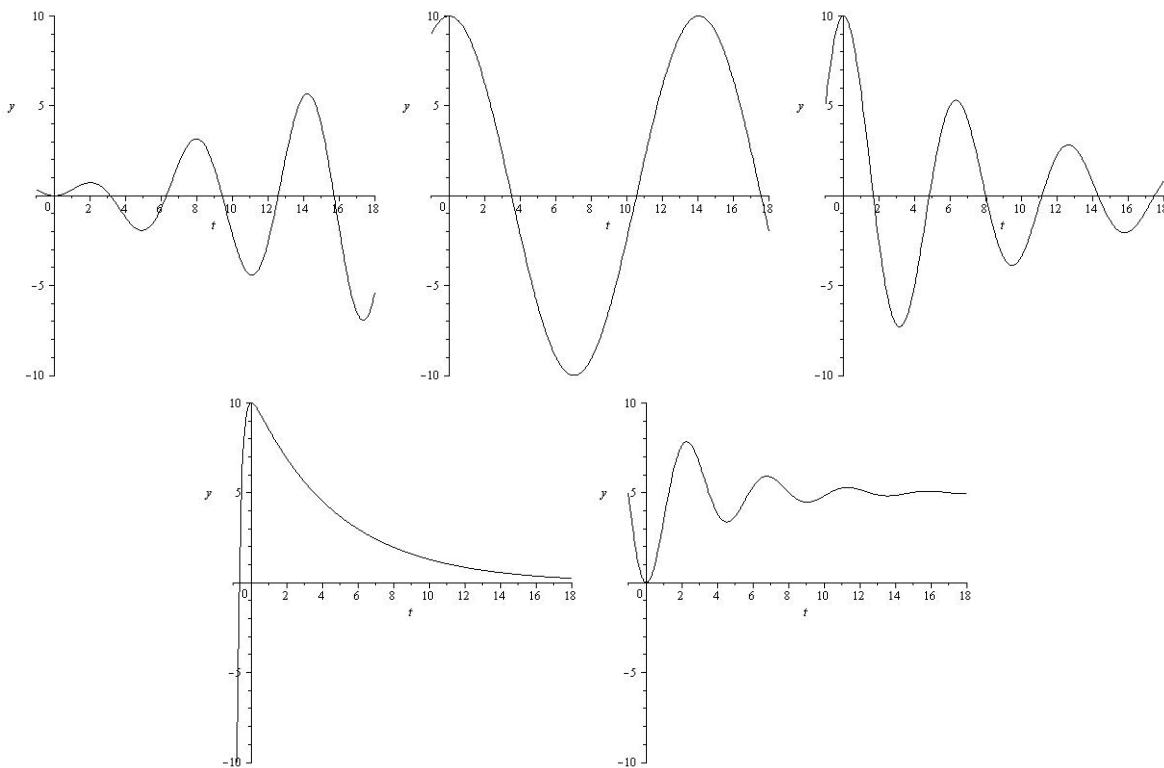
(a)  $5y'' + y' + 5y = 0, y(0) = 10, y'(0) = 0$

(b)  $y'' + 5y' + y = 0, y(0) = 10, y'(0) = 0$

(c)  $y'' + y' + \frac{5}{4}y = 0, y(0) = 10, y'(0) = 0$

(d)  $5y'' + 5y = 4 \cos(t), y(0) = 0, y'(0) = 0$

(e)  $y'' + \frac{1}{2}y' + 2y = 10, y(0) = 0, y'(0) = 0$



21. Be sure that you understand all of the homework problems from the Section 3.8 hand-out.