Math 244: Homework To Replace 7.1

1. Suppose we have two populations, $x(t), y(t)$. Three systems of differential equations are given below.

$$(a)\quad \begin{aligned} x' &= x(1-x) - xy \\ y' &= y(0.75 - y) - 0.5xy \end{aligned} \qquad (b)\quad \begin{aligned} x' &= x(1-x) - xy \\ y' &= -0.75y + xy \end{aligned}$$

$$(c)\quad \begin{aligned} x' &= x(1-x) - xy \\ y' &= -0.75y + xy \end{aligned}$$

   One of the models represents a predator-prey system. One of the models represents two species that are cooperating with each other, and one of the models represents two species competing with other.

   Which model could go with which system of DEs? (and explain)

2. Consider the two systems below (nonlinear)

$$\begin{aligned} x' &= 10x(1 - 0.1\,x) - 20xy \\ y' &= -5y + 0.05xy \end{aligned} \qquad \begin{aligned} x' &= 0.3x - 0.01\,xy \\ y' &= 15y(1 - 0.05y) + 25xy \end{aligned}$$

   In one of these systems, the prey are very large animals and the predators are very small animals, like elephants and mosquitos. Thus it takes many predators to eat one prey, but each prey eaten is a tremendous benefit for the predators. Determine which system fits this model the best.

3. Exercise 22 (Section 7.1, p 363, tank mixing)

4. Solve the system of equations given by first converting it into a second order linear ODE (then use Chapter 3 methods):

(a) $\begin{aligned} x' &= -2x + y \\ y' &= x - 2y \end{aligned}$ (b) $\begin{aligned} x' &= 2y \\ y' &= -2x \end{aligned}$

5. Convert the following second order differential equations into a system of autonomous, first order equations. Using methods from Chapter 3, give the solution to the system.

(a) $y'' + 4y' + 3y = 0$ (c) $y'' + 4y = 0$

(b) $y'' + 5y' = 0$ (d) $y'' - 2y' + y = 0$

6. Give the solution to each system of equations. If it has an infinite number of solutions, give your answer in vector form:

$$\begin{aligned} 3x + 2y &= 1 \\ 2x - y &= 3 \end{aligned} \qquad \begin{aligned} 3x + 2y &= 1 \\ 6x + 4y &= 3 \end{aligned} \qquad \begin{aligned} 3x + 2y &= 1 \\ 6x + 4y &= 2 \end{aligned}$$