HW- Graphical analysis and Poincare Classification

1. The matrices below represent the matrix A in a linear system, $\mathbf{x}' = A\mathbf{x}$. Classify the equilibrium (the origin) using the Poincaré Classification:

   (a) $\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -1 \\ 0 & -\frac{1}{4} \end{bmatrix}$

   (b) $\begin{bmatrix} -\frac{1}{2} & 1 \\ -1 & -\frac{1}{2} \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$

2. Discuss how the classification of the origin changes with α, given the matrix below.

   (a) $\begin{bmatrix} \alpha & -1 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \alpha & \alpha \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} \alpha & 1 \\ \alpha & \alpha \end{bmatrix}$

3. Suppose we are given $\mathbf{x}' = A\mathbf{x}$, and we compute the eigenvalues and eigenvectors (below). Draw a sketch of the phase plane in each case.

   (a) $\lambda_1 = -1, \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ with $\lambda_2 = 2, \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (b) $\lambda_1 = -1, \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ with $\lambda_2 = -2, \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (c) $\lambda_1 = 1, \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ with $\lambda_2 = 2, \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

4. Each matrix below represents the matrix A in the system $\mathbf{x}' = A\mathbf{x}$. For each, (i) first determine the eigenvalues/ eigenvectors (if needed), (ii) write down the general solution, and (iii) draw a sketch of the phase plane (the (x_1, x_2) plane).

   (a) $\begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$