Review Questions, Exam 1, Math 244

These questions are presented to give you an idea of the variety and style of question that will be on the exam. It is not meant to be exhaustive, so be sure that you understand the homework problems and quizzes.

1. Solve. If there is an initial condition, solve the IVP.

   (a) $y' = \dfrac{x^2 - 2y}{x}$
   Linear: $y' + \frac{2}{x}y = x$. The IF is $e^{\int 2/x\,dx} = e^{2\ln(x)} = x^2$. Multiply by the IF and continue:

   $$(x^2y)' = x^3 \to x^2y = \frac{1}{4}x^4 + C \quad \Rightarrow \quad y = \frac{1}{4}x^2 + \frac{C}{x^2}$$

   (b) $y' = \dfrac{2t + y}{3 + 3y^2 - t} \quad y(0) = 0.$
   This is not linear and not separable, so we check to see if it is exact:

   $$-(2t + y) + (3 + 3y^2 - t)y' = 0 \quad \Rightarrow \quad M_y = -1 = N_t \quad \Rightarrow \text{It is exact.}$$

   The solution is found by integrating M with respect to t or N with respect to y (or both). We'll start with N:

   $$f(t, y) = \int N\,dy = 3y + y^3 - ty + g(t)$$

   where g is an unknown function of t. We can determine g by finding f_t and compare to M:

   $$f_t = -y + g'(t) = -2t - y \quad \Rightarrow \quad g'(t) = -2t \quad \Rightarrow \quad g(t) = -t^2$$

   So far we see that $f(t, y) = 3y + y^3 - ty - t^2 = C$ is the solution. With $y(0) = 0$, we see that $C = 0$ giving the (implicit) solution:

   $$-t^2 - ty + 3y + y^3 = 0$$

   (c) $y' = -\dfrac{2xy + y^2 + 1}{x^2 + 2xy}$
   This is not linear and not separable, so we see if it is exact:

   $$(2xy + y^2 + 1) + (x^2 + 2xy)y' = 0 \quad \Rightarrow \quad M_y = 2x + 2y = N_x \quad \text{It is exact.}$$

   To go backwards from the gradient ($M = f_x, N = f_y$), we can integrate M with respect to x:

   $$f(x, y) = \int M\,dx = x^2y + xy^2 + x + g(y)$$

   We can either integrate N with respect to y and compare, or take our f and see if we get N:

   $$f_y(x, y) = x^2 + 2xy + g'(y) = x^2 + 2xy \quad \Rightarrow \quad g'(y) = 0$$

   Therefore, $f(x, y) = k$ is our (implicit) solution: $x^2y + xy^2 + x = C$.

   (d) $y' = 2\cos(3t) \quad y(0) = 2$
   This is linear and separable. $y(t) = \frac{2}{3}\sin(3t) + 2$.

   (e) $y' - \frac{1}{2}y = 0 \qquad y(0) = 200.$
   This is linear and separable. The solution is $y(t) = 200e^{(1/2)t}$

(f) $y' = (1-2x)y^2 \qquad y(0) = -1/6.$
This is separable.

$$\int \frac{dy}{y^2} = \int (1-2x)\,dx \quad \to \quad -\frac{1}{y} = x - x^2 + C \quad \to \quad y = \frac{1}{x^2 - x - 6}$$

(g) $y' - \frac{1}{2}y = \mathrm{e}^{2t} \qquad y(0) = 1$
As a linear DE, compute the integrating factor and multiply both sides by it: $\mathrm{e}^{-t/2}$

$$\mathrm{e}^{-t/2}y = \int \mathrm{e}^{3t/2}\,dt = \frac{2}{3}\mathrm{e}^{3t/2} + C$$

Therefore the general solution is the following, then solve for C:

$$y(t) = \frac{2}{3}\mathrm{e}^{2t} + C\mathrm{e}^{t/2} \quad \Rightarrow \quad 1 = \frac{2}{3} + C$$

The solution is:

$$y(t) = \frac{2}{3}\mathrm{e}^{2t} + \frac{1}{3}\mathrm{e}^{t/2}$$

(h) $y' = y(3-y)$
This is separable (and autonomous). When we separate variables and integrate, we'll need to use "partial fractions":

$$\frac{1}{y(3-y)} = \frac{A}{y} + \frac{B}{3-y} \quad \Rightarrow \quad 1 = A(3-y) + By$$

This equation must be true for all y, so some convenient ones are $y = 0$ and $y = 3$, from which we get that $A = 1/3$ and $B = 1/3$ so that

$$\int \frac{1}{y(3-y)}\,dy = \int \frac{1}{3}\frac{1}{y}\,dy + \frac{1}{3}\int \frac{1}{3-y}\,dy = \frac{1}{3}\ln|y| - \frac{1}{3}\ln|3-y|$$

Now back to the problem. On the left side we'll have integrated y, and on the right side, we integrate in t:

$$\frac{1}{3}\ln|y| - \frac{1}{3}\ln|3-y| = t + C \quad \Rightarrow \quad \ln\left|\frac{y}{3-y}\right| = 3t + C_2 \quad \Rightarrow \quad \frac{y}{3-y} = A\mathrm{e}^{3\mathrm{t}}$$

Solving for y, we get either:

$$y(t) = \frac{3A\mathrm{e}^{3t}}{1 + A\mathrm{e}^{3t}} = \frac{3}{1 + B\mathrm{e}^{-3t}}$$

(i) $\sin(2t)\;dt + \cos(3y)\;dy = 0$
Separable: $\int \cos(3y)\,dy = -\int \sin(2t)\,dt$, or $\frac{1}{3}\cos(3y) = \frac{1}{2}\cos(2t) + C$ (implicit form).

(j) $y' = xy^2$
Separable:

$$\int \frac{1}{y^2}\,dy = \int x\,dx \quad \Rightarrow \quad -\frac{1}{y} = \frac{1}{2}x^2 + C = \frac{x^2 + C_2}{2} \quad \Rightarrow \quad y(x) = \frac{-2}{x^2 + C_2}$$

(k) $y' + 2y = g(t)$, $y(0) = 0$ and

$$g(t) = \begin{cases} 1 & \text{if } 0 \le t \le 1 \\ 0 & \text{if } t > 1 \end{cases}$$

SOLUTION: This is similar to Exercise 33, Section 2.4. In this case, we go ahead and solve starting at time 0:

$$y' + 2y = 1 \quad \Rightarrow \quad y(t) = \frac{1}{2} - \frac{1}{2}\mathrm{e}^{-2t}$$

and this is valid for $0 \le t \le 1$. When we hit $t = 1$, the dynamics change to:

$$y' + 2y = 0 \quad \Rightarrow \quad y(t) = P\mathrm{e}^{-2t}$$

Now we will typically choose the constants so that y is continuous. Therefore, using our previous function, $y(1) = (1 - \mathrm{e}^{-2})/2$, and our current function: $y(1) = P\mathrm{e}^{-2}$, or

$$P = \frac{e^2 - 1}{2}$$

Therefore, the overall solution to the DE would be:

$$y(t) = \begin{cases} (1 - e^{-2t})/2 & \text{if } 0 \le t \le 1 \\ ((e^2 - 1)/2)\mathrm{e}^{-2t} & \text{if } 1 > 1 \end{cases}$$

2. Substitution:

(a) Show that the DE becomes separable with the change of variables: $v = y/x$: $(x+y)\,dx - (x-y)\,dy = 0$.

SOLUTION: We'll need to substitute something in for dy/dx in terms of v, x.

$$y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now we substitute:

$$(x-y)\,\frac{dy}{dx} = x+y \quad \Rightarrow \quad (x-vx)\,(v - xv') = x+vx \quad \Rightarrow \quad v - xv' = \frac{1+v}{1-v} \quad \Rightarrow \quad -xv' = \frac{1+v}{1-v} - v$$

And we can see that this is separable.

(b) Show that the DE becomes linear in v with the change of variables: $v = y^2$: $\frac{dy}{dx} - \frac{3}{2x}y = \frac{2x}{y}$

SOLUTION: Using the suggested substitution, we find an expression for the derivative:

$$\frac{dv}{dx} = 2y\frac{dy}{dx} \quad \text{or} \quad \frac{dy}{dx} = \frac{1}{2y}\frac{dv}{dx}$$

Substitute everything in now. I note that we don't currently have y^2 in the expression, so we'll multiply through by y first:

$$y\frac{dy}{dx} - \frac{3}{2x}y^2 = 2x \quad \Rightarrow \quad y\left(\frac{1}{2y}\frac{dv}{dx}\right) - \frac{3}{2x}v = 2x \quad \Rightarrow \quad \frac{dv}{dx} - \frac{3}{x}v = 4x$$

And this is linear in v.

(c) Let $p(y) = y'$, and rewrite the DE below in terms of p, y alone. $yy'' = (y')^2$ (Hint: Let $p(y) = y'$, and see if you can get the equation in terms of p and y).

SOLUTION:

$$\frac{dy}{dt} = p(y) \quad \Rightarrow \quad \frac{d^2y}{dt^2} = \frac{dp}{dy}\cdot\frac{dy}{dt} = \frac{dp}{dy}p(y)$$

Now, substitute in the expressions and divide by $yp(y)$:

$$y\frac{dp}{dy}p(y) = (p(y))^2 \quad \Rightarrow \quad \frac{dp}{dy} = \frac{1}{y}p$$

This equation is now separable (in p and y).

3. True or False, and explain:

   (a) If $y' = y + 2t$, then $0 = y + 2t$ is an equilibrium solution.

   False: This is an isocline associated with a slope of zero, and furthermore, $y = -2t$ is not a solution, and it is not a constant. Also, $y = -2t$ is not constant...

   (b) Let $\frac{dy}{dt} = 1 + y^2$. The Existence and Uniqueness theorem tells us that the solution (for any initial value) will be valid for all t. (If true, say why. If False, solve the DE).

   False. The E & U Theorem tells that a unique solution will exist for any initial condition (since $1 + y^2$ and $2y$ are continuous everywhere), but it does not say on what interval the solution will exist. For example, if we take $y(0) = y_0$ and solve, we get:

$$\int \frac{dy}{1+y^2} = \int dt \quad \Rightarrow \quad \tan^{-1}(y) = t + C \quad \Rightarrow \quad C = \tan^{-1}(y_0)$$

   Therefore,

$$y = \tan(t + \tan^{-1}(y_0))$$

   where

$$-\frac{\pi}{2} < t + \tan^{-1}(y_0) < \frac{\pi}{2}$$

   (so that the tangent function is invertible).

   (c) If $y' = \cos(y)$, then the solutions are periodic.

   FALSE. A function y is periodic if it is periodic in t, and once a function increases (for example), it cannot decrease again (since the slopes along any horizontal line are constant).

   (d) All autonomous equations are separable.

   True. Any autonomous equation can be written as $y' = f(y) \cdot 1$, which is separable and $\int \frac{dy}{f(y)} = \int dt$.

   (e) All separable equations are exact.

   True. If the equation is separable, then $y' = f(y)g(x)$, which can be written:

$$\frac{1}{f(y)} \frac{dy}{dx} = g(x) \quad \Rightarrow \quad -g(x) + \frac{1}{f(y)} \frac{dy}{dx} = 0$$

   Now, if $M(x, y) = -g(x)$, then $M_y = 0$, and $N(x, y) = 1/f(y)$ means $N_x = 0$.

4. Let $y' = y^{1/3}$, $y(0) = 0$. Find two solutions to the IVP. Does this violate the Existence and Uniqueness Theorem?

   To solve $y' = y^{1/3}$, we separate variables:

$$y^{-1/3}\, dy = dt$$

   Before going further, it is good practice to note that the previous step is valid, *as long as* $y \neq 0$. The case that $y = 0$ can be taken separately- In fact, we see that $y(t) = 0$ is an equilibrium solution that satisfies the initial condition.

   Going on, we integrate:

$$\frac{3}{2} y^{2/3} = t + C_1 \quad \Rightarrow \quad y^{2/3} = \frac{2}{3} t + C_2 \quad \Rightarrow \quad y = \left(\frac{2}{3} t + C_2\right)^{3/2}$$

We can solve for C_2 using the initial condition: $0 = C_2$, so that

$$y = \left(\frac{2t}{3}\right)^{3/2}$$

We can verify that this is indeed a solution by substituting it back into the DE (not necessary; just a way of double-checking yourself). This does not contradict the E&U Theorem, since $f_y = \frac{1}{3}y^{-2/3}$, so we would need to stay away from $y = 0$ for this to be continuous.

5. Let $(t+1)(t-2)y' + 3ty = \mathrm{e}^t$, with $y(1) = 2$. What is the largest interval on which we can guarantee the existence of a unique solution? (Hint: Do NOT solve the IVP).

   Solution: Write in standard form:

$$y' + \frac{3t}{(t+1)(t-2)}y = \frac{\mathrm{e}^t}{(t+1)(t-2)}$$

   Our functions $p(t)$ and $g(t)$ are each continuous on either $(-\infty, -1)$, or $(-1, 2)$, or $(2, \infty)$. Because $y(1) = 2$, choose the interval containing $t = 1$, or $(-1, 2)$.

6. If $\frac{dy}{dx} = \frac{x-3}{(x-1)(y-2)}$, then

   (a) find the areas of the plane that our initial value must avoid in order to (be explicit in your computations!), and

   (b) if $y(0) = 1$, is it possible to predict the interval on which the solution exists?

   For the first part, $f(x, y) = \frac{x-3}{(x-1)(y-2)}$ so for f to be continuous, we must stay away from the vertical line $x = 1$ and the horizontal line $y = 2$. Further, we check f_y:

$$f_y = \frac{x-3}{x-1}(-(y-2)^{-2})$$

   and we don't gain any additional restrictions. We see that if $y(0) = 1$, then we are indeed guaranteed that a unique solution exists, but we cannot predict in advance what the interval (in x) will be- We would have to solve the IVP.

7. Find a constant a and a function $g(t)$ if $y(t)$ is to be a solution to the DE: $y' + ay = g(t)$.

   (a) $y(t) = t - 3 + C\mathrm{e}^{3t}$
   Using this y, we see what $y' + ay$ is:

$$y' + ay = (1 + 3C\mathrm{e}^{3t}) + a(t - 3 + C\mathrm{e}^{3t})$$

   We want to be sure that $y' + ay$ does not contain the arbitrary constant C, so we see that $a = -3$. With this simplification:

$$y' - 3y = 1 + 3C\mathrm{e}^{3t} - 3t + 9 - 3C\mathrm{e}^{3t} = 3t + 10$$

   So $a = -3$ and $g(t) = 3t + 10$.

   (b) $y(t) = 2\sin(3t) + C\mathrm{e}^{-2t}$
   SOLUTION: Try to get y' and ay together in such a way as to cancel out the arbitrary C:

$$y' = 6\cos(3t) - 2C\mathrm{e}^{-2t}$$

   so that: $y' + 2y = 4\sin(3t) + 6\cos(3t)$.

8. Suppose we want to construct a population model using our logistic model. Assuming the initial population growth rate is 0.1, and the environmental threshold (or carrying capacity) is 100, write down the model. Using your model, now assume there is a continuous"harvest" (of k units per time period). How does that effect the model- In particular, is there a critical value of k over which the population will be extinct? If so, find it.

   SOLUTION: The initial logistic equation would be:

$$y' = \frac{1}{10}y(\left(1 - \frac{y}{100}\right)$$

   With the continuous harvest, the model becomes:

$$y' = \frac{1}{10}y(\left(1 - \frac{y}{100}\right) - k$$

   Geometrically, the upside parabola is being shifted down by k units. One way to solve this analytically is to realize that, for some value of k, the parabola will touch the y axis at only one single point- At that point, the parabola will be a perfect square.

$$\frac{1}{10}y(1 - y/100) - k = \frac{-1}{1000}\left(y^2 - 100y + 1000k\right)$$

   This expression will be a perfect square only when

$$1000k = 50^2 \quad \Rightarrow \quad k = \frac{5}{2}$$

   Now, when $k = 5/2$, note that the parabola becomes: $-\frac{1}{1000}(y - 50)^2$, which has only one equilibrium solution. If k becomes any bigger, the parabola drops below the y axis and we have no real equilibria.

9. Construct an autonomous differential equation with three equilibria: $y = -2, y = 0, y = 1$, and the two outer equilibria are stable, the inner equilibrium is unstable. (HINT: Start with a sketch in the (y, y') plane).

   SOLUTION: From the sketch, you should see that the polynomial is a cubic, which goes to minus infinity as y goes to positive infinity:

$$y' = -(y + 2)y(y - 1)$$

10. Suppose we have a tank that contains M gallons of water, in which there is Q_0 pounds of salt. Liquid is pouring into the tank at a concentration of r pounds per gallon, and at a rate of γ gallons per minute. The well mixed solution leaves the tank at a rate of γ gallons per minute.

    Write the initial value problem that describes the amount of salt in the tank at time t, and solve:

$$\frac{dQ}{dt} = r\gamma - \frac{\gamma}{M}Q, \qquad Q(0) = Q_0$$

    The solution (with the initial condition) is given by:

$$Q = rM + (Q_0 - rM)\mathrm{e}^{-(\gamma/M)t}$$

11. Referring to the previous problem, if let let the system run infinitely long, how much salt will be in the tank? Does it depend on Q_0? Does this make sense?

    The amount of salt goes to equilibrium, which would be M gallons times r lbs per gallon, or rM pounds of salt.

12. Modify problem 5 if: $M = 100$ gallons, $r = 2$ and the input rate is 2 gallons per minute, and the output rate is 3 gallons per minute. Solve the initial value problem, if $Q_0 = 50$.

$$\frac{dQ}{dt} = 4 - \frac{3}{100-t}Q \qquad Q(0) = 50$$

This goes from being autonomous to linear. In this case, use an integrating factor,

$$e^{\int p(t)\,dt} = e^{3\int \frac{1}{100-t}\,dt} = e^{-3\ln|100-t|} = (100+t)^{-3} \qquad t > 100$$

Going back to the DE

$$\left(\frac{Q}{(100-t)^3}\right) = 4(100-t)^{-3} \quad \Rightarrow \quad Q = 2(100-t) + C(100-t)^3$$

Continuing, we get:

$$Q(t) = 2(100-t) - \frac{150}{100^3}(100-t)^3$$

13. Suppose an object with mass of 1 kg is dropped from some initial height. Given that the force due to gravity is 9.8 meters per second squared, and assuming a force due to air resistance of $\frac{1}{2}v$, find the initial value problem (and solve it) for the velocity at time t. In the (t, y) plane, draw several solution curves (Hint: This is an autonomous DE).

SOLUTION: The general model is: $mv' = mg - kv$. In this case, $m = 1$, $g = 9.8$ and $k = 1/2$. Therefore,

$$v' = 9.8 - \frac{1}{2}v$$

Which is linear (and autonomous). Since the object is being dropped, the initial velocity is zero.

Solve it:

$$v(t) = 19.6\left(1 - e^{-(1/2)t}\right)$$

14. Suppose you borrow \$10000.00 at an annual interest rate of 5%. If you assume continuous compounding and continuous payments at a rate of k dollars per month, set up a model for how much you owe at time t in years. Give an equation you would need to solve if you wanted to pay off the loan in 10 years.

SOLUTION: First, notice that the units of time are mixed- The interest rate is an annual rate, but k is in dollars per month. We should first decide on what the units of time should be. The answer below will assume that we are working with time in *years*, so that the annual payments are $12k$ dollars per year (in a continuous fashion).

Therefore, the model is $S' = rS - 12k$, where S will be the amount owing, r is the annual interest rate and k is the rate for the continuous payment (per month). Then using $S(0) = S_0$, we can write the solution as

$$S(t) = \frac{12k}{r} + \left(S_0 - \frac{12k}{r}\right)e^{rt}$$

Substituting in the values for r and S_0, and $t = 10$, we can solve the equation for k:

$$0 = 240k + (10000 - 240k)e^{\frac{1}{2}}$$

Extra: If we go ahead and solve, we get a monthly payment rate of $k \approx \$105.90$.

15. Show that the IVP $xy' = y - 1$, $y(0) = 2$ has no solution. (Note: Part of the question is to think about how to show that the IVP has no solution. You might start by actually trying to solve it).

SOLUTION: We'll try to solve it, and see what happens. The DE is separable. Notice that the function $f(x, y)$ from the E& U theorem is $(y-1)/x$, which is not continuous at $x = 0$...

Continuing as usual:

$$\int \frac{1}{y-1}\,dy = \int \frac{1}{x}\,dx \qquad \Rightarrow \qquad \ln|y-1| = \ln|x| + C$$

Exponentiate both sides to get

$$y - 1 = Ax \qquad \Rightarrow \qquad y = Ax + 1$$

There is no choice of A that will satisfy the initial condition,

$$2 = A \cdot 0 + 1$$

16. Suppose that a certain population grows at a rate proportional to the square root of the population. Assume that the population is initially 400 (which is 20^2), and that one year later, the population is 625 (which is 25^2). Determine the time in which the population reaches 10000 (which is 100^2).

SOLUTION: If $P(t)$ is the population at time t, then the first part of the statement translates to:

$$\frac{dP}{dt} = kP^{1/2}$$

where k is the constant of proportionality. This is separable (and autonomous), so:

$$\int P^{-1/2}\,dP = \int k\,dt \qquad \Rightarrow \qquad P^{1/2} = (k/2)t + C$$

Given the initial population, we can solve for C, given the second piece of info, we can solve for k:

$$P(0) = 20^2 \qquad \Rightarrow \qquad 20 = (k/2)(0) + C \qquad \Rightarrow \qquad C = 20$$

and $P(1) = 25^2$ gives:

$$25 = (k/2) + 20 \qquad \Rightarrow \qquad k = 10$$

Therefore, our model is:

$$P(t) = (5t + 20)^2$$

Solving for the last part, $P(t) = 100^2$, we have

$$100 = 5t + 20 \qquad \Rightarrow \qquad t = 16$$

17. Consider the sketch below of $F(y)$, and the differential equation $y' = F(y)$.

(a) Find and classify the equilibrium.

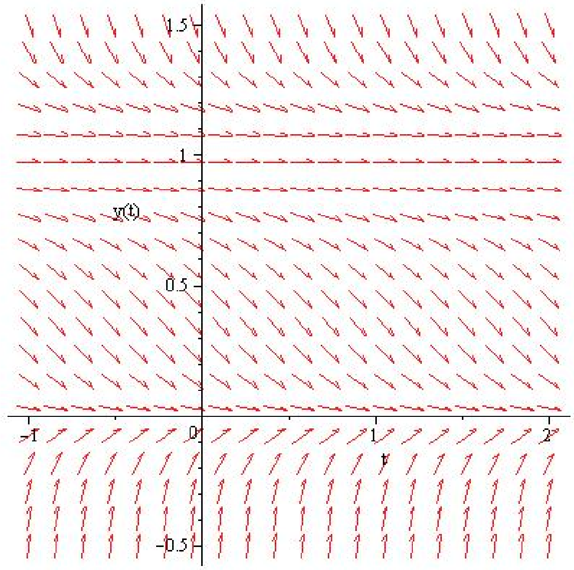
SOLUTION: From the sketch given, $y = 0$ is asymptotically stable and $y = 1$ is semistable.

(b) Find intervals (in y) on which $y(t)$ is concave up.

SOLUTION: Examine the intervals $y < 0$, $0 < y < 1/3$, $1/3 < y < 1$ and $y > 1$ separately. The function y will be concave up when dF/dy and F both have the same sign- This happens when F is either increasing and positive (which happens nowhere) or decreasing and negative:

$$0 < y < \frac{1}{3} \qquad y > 1$$

(c) Draw a sketch of y on the direction field, paying particular attention to where y is increasing/decreasing and concave up/down.



(d) Here is one example of the polynomial:

$$y' = -y(y-1)^2$$

18. Evaluate the following integrals:

$$\int x^3 \mathrm{e}^{2x}\,dx \qquad \int \frac{x}{(x-1)(2-x)}\,dx \qquad \mathrm{e}^{-3\int dt/t}$$

SOLUTIONS: You should be able to integrate by parts and use partial fractions fairly quickly at this point. Try checking your answers using Maple, Mathematica, or Wolfram Alpha.

19. Consider the DE: $y\,dx + (2x - y\mathrm{e}^y)\,dy = 0$. Show that the equation is not exact, but becomes exact if you assume there is an integrating factor in terms of y alone (Hint: Find the integrating factor first).

SOLUTION: Let μ be the integrating factor, and assume μ is a function of y alone. Then

$$(\mu M)_y = \mu' \cdot M + \mu \cdot M_y = y\mu' + \mu$$

And

$$(\mu N)_x = 0 \cdot N + \mu \cdot 2$$

Setting these equal, we have the DE:

$$y\mu' = \mu \quad \Rightarrow \quad \int \frac{1}{\mu}\,d\mu = \int \frac{dy}{y}$$

or $\mu = y$. We can verify our answer, since $y^2\,dx + (2xy - y^2 e^y)\,dy = 0$ should now be exact.

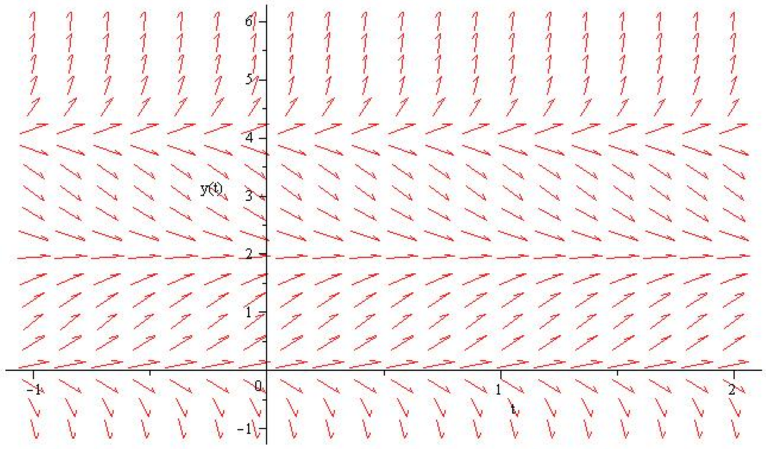
20. Newton's Law of Cooling states that the rate of change of the temperature of a body is proportional to the difference between the temperature of the body and the environment (which we assume is some constant). Write down a differential equation which represents this statement, then find the general solution.

SOLUTION: If $u(t)$ is the temperature of the body at time t, and T is the (constant) environmental temperature (please define your terms!), then we have:

$$u' = -k(u - T) \quad \Rightarrow \quad u(t) = C\mathrm{e}^{-kt} + T$$

This solution makes sense, since the long term behavior is that the temperature reaches T as $t \to \infty$.

21. Given the direction field below, find a differential equation that is consistent with it.



SOLUTION: Draw the corresponding figure in the (y, y') plane first. There we see that $y = 0$ is unstable (make it a linear crossing), and $y = 2$ is stable, $y = 4$ is unstable. From the figure,

$$y' = y(y-2)(y-4)$$

will work.

22. Consider the direction field below, and answer the following questions:

   (a) Is the DE possibly of the form $y' = f(t)$?
   SOLUTION: No. The isoclines would be vertical (consider, for example, a vertical line at $t = -3$; the slopes are clearly not equal).

   (b) Is the DE possible of the form $y' = f(y)$?
   SOLUTION: No. The isoclines would be horizontal (for example, look at a horizontal line at $y = 1$- Some slopes are zero, others are not).

   (c) Is there an equilibrium solution? (If so, state it):
   SOLUTION: Yes- At $y = 0$.

   (d) Draw the solution corresponding to $y(-1) = 1$.
   SOLUTION: Just draw a curve consistent with the arrows shown.