L003 Section 6.1 Examples, part 1 notes

Note: I usually print out and distribute a table of transforms, but there is one in the text that you can usue- Table 6.2.1 on page 317 (Section 6.2).

Some Computational Examples, post 6.1

- Find the Laplace transform for $t^2 + 2t + 3\sin(2t)$ using the properties of the transform and the table of transforms.

  SOLUTION:

  $$\mathcal{L}(t^2+2t+3\sin(2t)) = \mathcal{L}(t^2)+2\mathcal{L}(t)+3\mathcal{L}(\sin(2t) = \frac{2}{s^3}+2\frac{1}{s^2}+3\frac{2}{s^2+4}$$

- Use the table of transforms to invert the Laplace transform:

  $$\mathcal{L}^{-1}\left(\frac{4}{s-1}\right) = 4\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) = 4\mathrm{e}^t$$

- Find the inverse transform of the given expression:

  $$\frac{3s}{s^2-s-6} = \frac{3s}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$$

  Use partial fractions to see that $A = 6/5$ and $B = 9/5$ (details on the video if you need a refresher).

  $$\frac{3s}{s^2-s-6} = \frac{6}{5}\cdot\frac{1}{s+2} + \frac{9}{5}\cdot\frac{1}{s-3}$$

  Taking the inverse transform, we get

  $$\frac{6}{5}\mathrm{e}^{-2t} + \frac{9}{5}\mathrm{e}^{3t}$$

- Find the inverse Laplace transform:

  $$\frac{8s^2-4s+12}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4}$$

  Doing the partial fractions,

  $$8s^2-4s+12 = A(s^2+4)+(Bs+C)s = (A+B)s^2+Cs+4A$$

  From this, we get three equations by equating the coefficients of the polynomials on the left and the right:

  $$\begin{array}{lrl} s^2: & 8 & = A+B \\ s: & -4 & = C \qquad \Rightarrow \qquad A=3, B=5, C=-4 \\ const: & 12 & = 4A \end{array}$$

Therefore,

$$\frac{8s^2 - 4s + 12}{s(s^2+4)} = \frac{3}{s} + \frac{5s-4}{s^2+4} = 3\frac{1}{s} + 5\frac{s}{s^2+4} - 2\frac{2}{s^2+4}$$

Now we can invert each piece:

$$3 + 5\cos(2t) - 2\sin(2t)$$