# Group Practice

Find the general form of the solution to the corresponding differential equation:

1. $4y''+4y'+y=0$ Complete the square: $4r^2+4r+1=0$ so $4(r^2+r+1/4)=4(r+1/2)^2=0$ (OR: $(2r+1)^2=0$), and the general solution is$y=\mathrm{e}^{-t/2}(C_1+C_2t)$

2. $y''+4y'+13=0$ Complete the square: $r^2+4r+13=0$ so $r^2+4r+4+9=0$, $(r+2)^2=-9$ so $r=-2\pm 3i$, and the general solution is $y=\mathrm{e}^{-2t}(C_1\cos(3t)+C_2\sin(3t))$

3. $y''+y'-2y=0$ Factor: $r^2+r-2=0$ so $(r+2)(r-1)=0$, so the general solution is $y=C_1\mathrm{e}^{-2t}+C_2\mathrm{e}^{t}$

4. $y''+2y'+y=0$ Factor: $r^2+2r+1=0$ so $(r+1)^2=0$, so the general solution is $\mathrm{e}^{-t}(C_1+C_2t)$

5. $3y''+2y'+y=0$ Complete the square or use the quadratic formula: $3r^2+2r+1=0$ $\Rightarrow \quad r=\frac{-1\pm\sqrt{2}\,i}{3}$ so the general solution is $y=\mathrm{e}^{-t/3}\left(C_1\cos\left(\frac{\sqrt{2}}{3}\,t\right)+C_2\sin\left(\frac{\sqrt{2}}{3}\,t\right)\right)$

# Find the second solution using $W$

Use the Wronskian to find the second solution, given that

$$t^2y''-4ty'+6y=0 \qquad y_1(t)=t^2$$

The idea is that we compute $W(y_1,y_2)(t)$ two ways (one way using the definition, the second using Abel's Theorem), then set them equal. This gives a differential equation for $y_2$ that we can solve.

$$W(y_1,y_2)=\begin{vmatrix} t^2 & y_2 \\ 2t & y_2' \end{vmatrix}=t^2y_2'-2ty_2 \qquad W(y_1,y_2)(t)=C\mathrm{e}^{-\int -4/t\,dt}=Ct^4$$

Therefore,

$$t^2y_2'-2ty_2=Ct^4 \quad \Rightarrow \quad y_2'-\frac{2}{t}y_2=Ct^2$$

The integrating factor is $t^{-2}$, so

$$(y_2/t^2)'=C \quad \Rightarrow \quad y_2/t^2=Ct+D \quad \Rightarrow \quad y_2=Ct^3+Dt^2$$

This is a linear combination of $t^2$ and $t^3$. We already have $t^2$, so we take $y_2=t^3$.