M244
Spr 2022

Quiz 4

Show all your work! This is a take home quiz. You may use your text and a calculator, but the work should be your own. Write your solutions up clearly, neatly and completely! Upload them to Canvas by Friday at 11:59PM.

1. Use the existence and uniqueness theorem to determine if (i) a solution to the IVP *exists*, and if existence is guaranteed, (ii) whether or not the soluton is *unique*.

   (a) $\dfrac{dy}{dx} = \sqrt{x-y}$ with $y(2) = 2$

   (b) $\dfrac{dy}{dx} = x\ln(y)$ with $y(1) = 1$

2. **Without solving the DE**, find the interval on which we can guarantee a unique solution exists (and give a short reason why).

   (a) $(4-t^2)y' + 2ty = 3t^2$ with $y(-3) = 1$

   (b) $(ln(t))y' + y = \cot(t)$ with $y(2) = 3$.

3. (Problem 28 in 2.4) The following equation is a Bernoulli equation. Solve it by using the suggested substitution in Problem 27, or as described in class.

$$t^2y' + 2ty - y^3 = 0, \quad t > 0$$

4. (Similar to #3, in 2.5) Given that $y' = f(y) = (y-1)(y-2)$,

   (a) Draw the graph of y' in the (y, y') plane, and locate the equilibrium solutions.

   (b) Classify each equilibrium (as asymptotically stable, unstable, or semistable).

   (c) Draw sample solution curves in the (t, y) plane (several curves are fine).

5. Write $\frac{1}{(y-1)(y-2)}$ using partial fractions (as if we were about to integrate).