Homework 2 solutions

1. (a) The equilibrium is found where $D(x) = S(x)$, which is the ordered pair

$$\left(\frac{11}{2}, \frac{9}{2}\right)$$

(b) If $q_1 = 7$, we get the following points for the market adjustments:

$$(7, 10-7) = (7,3), \quad (3 = x-1, 3) = (4,3), \quad (4, 10-4) = (4,6),$$

$$(6 = x-1, 6) = (7,6), \quad (7,3)$$

and we observe that we're back to the starting point. You should observe the same behavior for any initial point.

(c) For this part, I wanted you to think about what functions were being applied to the points. That is, let's do the same computations with a "generic" p_1:

$$(q_1, p_1) \quad \Rightarrow \quad \left(S^{-1}(p_1), p_1\right) \quad \Rightarrow \quad \left(S^{-1}(p_1), D(S^{-1}(p_1))\right) \quad \Rightarrow$$

$$\left(S^{-1}DS^{-1}(p_1), DS^{-1}(p_1)\right) \Rightarrow \left(S^{-1}DS^{-1}(p_1), DS^{-1}DS^{-1}(p_1)\right) \quad \Rightarrow \dots$$

Therefore, $p_2 = D(S^{-1}(p_1)) = f(p_1) = 9 - p_1$, and $f \circ f(p_1) = 9 - (9 - p_1) = p_1$

(d) i. If $S(x) = 3x - 1$, then the equilibrium is $(11/4, 29/4) = (2.75, 7.25)$
The orbit is (rounded, and including enough points to see if it is converging):

$$(7,3), (1.33, 3), (1.33, 8.66), (3.22, 8.66),$$

$$(3.22, 6.77), (2.59, 6.77), (2.59, 7.40), (2.80, 7.40),$$

$$(2.80, 7.19), \dots$$

These market fluctuations are converging, and you should observe the same behavior for any starting point.

ii. If $S(x) = \frac{1}{2}x - 1$, then the equilibrium is $(22/3, 8/3) \approx 7.33, 2.66)$
The orbit is (rounded, and including enough points to see if it is converging:):

$$(7,3), (8,3), (8,2), (6,2), (6,4), (10,4), \dots$$

These market fluctuations are getting bigger and bigger in magnitude, and you should see the same behavior with any starting point (unless you start at equilibrium).

2. For the problem from the text (2.4, p. 35), the data given suggests that the demand curve is the line between $(0.14, 0.986), (3, 0.7)$ and the supply curve is the line between $(0.14, 0.7), (0.1972, 0.986)$. Therefore, we find that:

$$D(x) = -\frac{1}{10}x + 1 \qquad S(x) = 5x$$

Using these functions, we fill in the table:

$$A = 0.1972,\ B = D(0.1972) = 0.98028,\ C = S^{-1}(0.98028) = 0.196056,\ D = 0.98029$$

The equilibrium is $\left(\frac{10}{51}, \frac{50}{51}\right) \approx (0.196, 0.98)$, and it looks like the market fluctations are converging to it.

3. Compute the sum from the notes (pg. 6):

$$V_b = \frac{V_N}{(\beta^2\gamma)^N} \cdot \frac{1 - (n\beta^2\gamma)^{N+1}}{1 - (n\beta^2\gamma)}$$

with the numbers given, $b^2\gamma = \frac{1}{18}$, $n = 3$, $N = 10$, and $V_{10} = \pi r_{10}^2 l_{10} = \pi$:

$$V_b = \pi \cdot 18^{10} \cdot \frac{1 - (1/6)^{11}}{1 - (1/6)}$$

Using Maple, that was about 1.34×10^{13}, or $4.28\pi \times 10^{12}$.

Using the approximation from the notes, it would be

$$V_b \approx \left(\frac{V_N}{1 - n\beta^2\gamma}\right) \cdot (\beta^2\gamma)^{-N} = \left(\frac{6\pi}{5}\right) \cdot 18^{10}$$

which gives the same answers to about 9 digits of accuracy.

NOTE: The β, γ in this example are not biologically plausible- In that case, we showed that $\gamma = n^{-1/3}$ and $\beta = n^{-1/2}$. Since the wording of the question did not specify that, I didn't count off if you used the summation formula involving only n.

4. Various responses possible.

5. Plot the data. Our conclusion: It is possible to have three small groupings within the larger group, where each of the three smaller sets each have a best line slope of $-1/3$, but taken together, have a best line slope of $-1/4$.

6. Kepler's Third Law: $P^2 \propto D^3$ (periods and distances from our data set). To verify Kepler's third law, we could write either:

$$P = kD^{3/2} \quad \Rightarrow \quad \ln(P) = \ln(k) + \frac{3}{2}\ln(D)$$

or

$$D = kP^{2/3} \quad \Rightarrow \quad \ln(D) = \ln(k) + \frac{2}{3}\ln(P)$$

So the slope of your line of best fit should have been either 3/2 or 2/3, depending on how you formulated the equation.