Case Study 1: "I Will If You Will.."

September 3, 2003

The material described herein was liberally adopted from (1) "I will if you will... Individual Thresholds and Group Behavior", J.S. Growney, 1987, UMAP Module 519, and (2) "Threshold Models of Collective Behavior", Mark Granovetter, American Journal of Sociology, 1978, v 83, n 6, 1421-1443.

1 The Tools

In this case study, we will use the techniques of modeling by functions and function iteration to see if group behaviors are predictable based on group norms. In this example, it may be difficult to get an analytic solution, but we can perform what's called a "graphical analysis" of the model to get predictions.

Before we start, let's consider the idea of "function iteration". This is the process of collecting a sequence of data (also called the *orbit* of x_0:

$$x_0, x_1, x_2, x_3, x_4, \ldots, x_T$$

where

$$x_{i+1} = f(x_i)$$

To get things started, we must be given an initial value, x_0. Note that using this definition, we could have written the orbit as:

$$x_0, f(x_0), f(f(x_0)), f(f(f(x_0))), f(f(f(f(x_0)))), \ldots$$

and thus the "iteration". Here are some short exercises to get you thinking about function iteration. In this example, let $f(x) = \sqrt{x}$.

- Use a calculator to compute the orbits using the following initial conditions (compute the first 10 or so):

  $$x_0 = 0 \quad x_0 = 0.2 \quad x_0 = 0.8 \quad x_0 = 1 \quad x_0 = 4$$

  Predict what the orbit will do as you increase the orbit length.

- Solve for x, if $x = \sqrt{x}$. These are called the *fixed points* for $f(x) = \sqrt{x}$, since iterating these will give constant orbits.

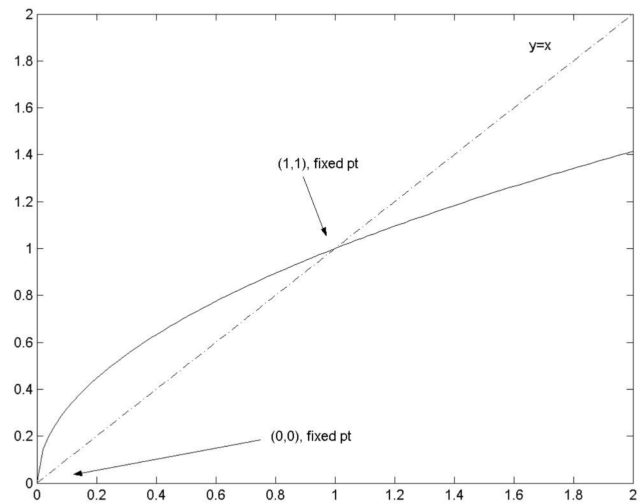

Figure 1: A plot of $y = x$ and $y = \sqrt{x}$. Where these curves intersect is where the fixed points are located. Here we have two fixed points, $x = 0$ and $x = 1$.

- A fixed point is *attracting* if orbits beginning nearby will eventually go to that fixed point. A fixed point is *repelling* if orbits beginning nearby move away from the fixed point. Classify the two fixed points found above as either attracting or repelling.

We can analyze the fixed points graphically by plotting two curves: $y = f(x)$ and $y = x$. Where these curves intersect will give us the locations of the fixed points. In Figure 1, we show a graph of $y = x$ and $y = \sqrt{x}$.

We can graphically determine the orbit of a point by doing the following:

- Go vertically to the graph. This gives us (x_0, x_1).
- Go horizontally to the line. This gives us (x_1, x_1).
- Go vertically to the graph. This gives us (x_1, x_2).
- Repeat.

This process is shown in Figure 2, where we see the values $x_0, x_1, x_2, \ldots$. The graphical analysis tells us that this orbit is approaching $x = 1$.

2 The Model

Let's begin with the following scenario: Suppose we have 100 people. One person will riot even if no others do (this is the *instigator*). One person will riot

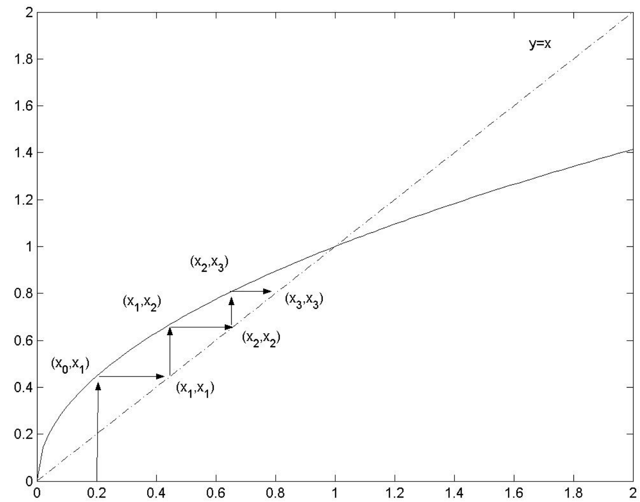

Figure 2: We can graphically determine the orbit of a point x_0 by going vertically to the graph, the horizontally to the line, then vertically to the graph, etc. Here, we see that the orbit of $x_0 = 0.2$ is moving away from $x = 0$ and towards $x = 1$.

if 1 other person does. One person will riot if 2 other people do. One person will riot if 3 others do, and so on. Will there be a riot? We see quickly that once the instigator begins, the next person will riot, since there is one other person doing it. The next person sees 2 people rioting, so they join in, and so on. We might call this the bandwagon, or domino effect.

Now change the distribution slightly, and remove the person with a threshold of 3 and make it a threshold of 4. Now, an entirely different process will occur- The instigator begins, the next 2 people join in, but now, since there are only 3 people rioting (which is below the next threshold), no one else will join in.

Before continuing, let us make our assumptions clear. In this model, we will assume that:

- Each "actor" has to make a decision- To do something or not to do something. These choices are distinct (like "riot" or "not riot", "cheat" or "not cheat", etc.).

- The decision is based on a threshold: How many people would have to do it before you will? There are some implications here- for example, we are not taking friendships into account (a person may be more likely to do something if friends are doing it, versus strangers).

- Lastly, we are assuming that the actor knows how many people are doing it. In reality, this would be the number that the actor only *perceives* doing

Threshold	Number with that Threshold	Number $\leq$ Threshold
0	0	0
10	15	15
20	15	30
30	10	40
40	10	50
50	30	80
60	10	90
70	0	90
80	0	90
90	0	90
100	0	90

Table 1: Results of a survey. The first column is the threshold value, rounded to the nearest 10. The second column is the number of people whose threshold is that value. The third column is the number of people whose threshold is at or below the first column value (it is a cumulative sum). For example, the third line tells us that there are 15 people with a threshold of 20, and there are 30 people altogether with thresholds less than or equal to 20.

it.

Now, let's examine how we set the model up with a particular example. Suppose I give a survey, with the results given in Table 1. You can think of the threshold values as being either the number of people or if there is more than 100, think of these as percentages.

Does a fixed point make sense in this example? Suppose there were 30 people with a threshold of 30 (or below). Then no one else would join in, and the number of people doing it would remain fixed (or, be at equilibrium). In our case, there are 40 people with a threshold of 30. Therefore, if all these people joined in at 30, there would be 40 people rioting. But, if all these people joined in, there are 50 people total with a threshold of 40 or below, so they would join in, and so on. What will happen in the long term? We will make a plot using the cumulative values to see. The plot is given in Figure 3.

Given these survey results, we can conclude that if there is a perception of anyone rioting, then our prediction will be that 90 people (or 90%) will riot.

Now suppose that the survey changes slightly. The numerical values are given in Table 2, and the corresponding cumulative plot is given in Figure 4. Can you predict what will happen now?

To get things started in these models, there is an initial perception (accurate or not) of how many people are doing it.

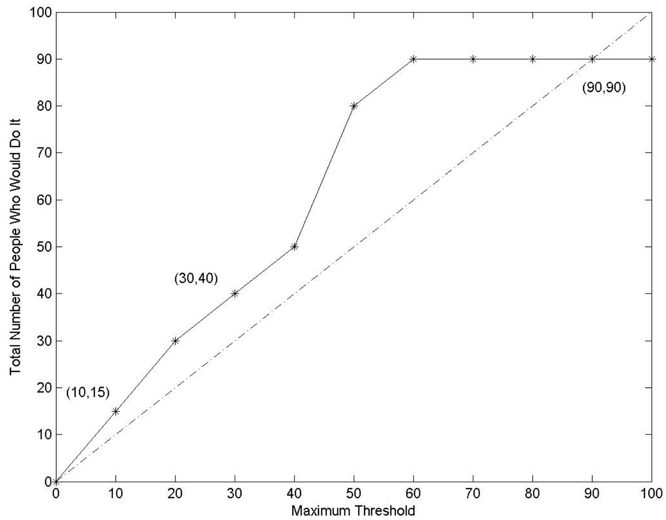

Figure 3: A cumulative plot of thresholds. Here we see the equilibrium at $(90, 90)$. We can trace the orbit of an initial condition (say, $x_0 = 10$) to see what the end result will be- at the end, there will be 90 people rioting. The point $(10, 15)$ for example means that there are a **total** of 15 people willing to riot if there are at least 10 already doing it.

Threshold	Number with that Threshold	Number $\leq$ Threshold
0	0	0
10	10	10
20	5	15
30	10	25
40	10	35
50	15	50
60	40	90
70	0	90
80	0	90
90	0	90
100	0	90

Table 2: Results of a second survey.

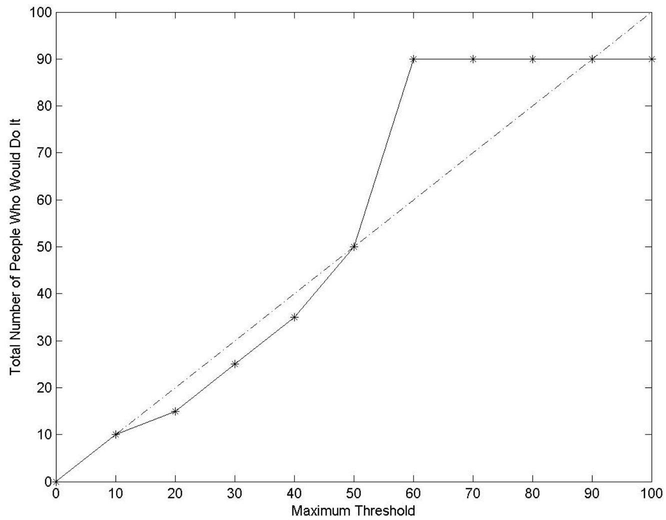

Figure 4: A cumulative plot of thresholds for the second survey. Can you predict the outcome(s)?

3 Group Project (if enough people)

A new fad is going around- people are wearing a cheese hat! Hypothetically (you don't need to be truthful!) how many people (or what percent of the students) would need to be wearing the cheese hat before you will?

Take an informal survey, and try to predict how many people will be wearing a cheese hat.

4 Exercises

1. As summer progresses toward autumn, many communities begin to worry about whether their water supply will last until it rains again. At such times, these localities may impose restrictions that limit the amount of water use. Such restrictions are hard to enforce, however. Furthermore, if one person is seen watering his lawn, his neighbors may wonder, "What good does it do me to conserve water when others don't?" and may violate the limits as well.

   (a) Suppose Walla Walla is such a town, and 90% support conservation if everyone else does. Invent data (graphically) to illustrate that, despite a 90% support of conservation, a situation can arise in which no one conserves.

Figure 5: A new(?) fad!

(b) Suppose you're the mayor of Walla Walla, and you see this situation occurring. What steps can you take to help the community achieve the conservation that most of them want?

2. The equilibria may not be a whole number- even when the numbers are not percentages, but the actual number of people. Here are data for two cases, where we give the number of people choosing certain thresholds (these are not cumulative numbers).

Case 1:

Threshold	0	20	40	60	80	100
Number	0	40	5	10	5	0

Case 2:

Threshold	0	20	40	60	80	100
Number	0	10	10	60	10	10

Do a graphical analysis in each case, and answer the following:

(a) Explain why the behavior in Case 1 can be expected to stabilize at 45.

(b) Explain why the behavior in Case 2 will stabilize either at 0 or 100.

(c) If a graph of threshold data has a slope less than 1 when it crosses equilibrium, will the behavior go towards or away from that equilibrium?

(d) Same question, but suppose the slope is greater than 1.