

Instructions: This is an open book, open notes exam, and you may use Maple and/or Matlab. You are expected to do your own work. The exam is due in my office (Olin 234) by Friday at noon.

Turn in everything you want me to consider- This can include a combination of handwritten notes, Matlab/Maple graphs, etc. If you're not sure, it's better to turn in more than less.

Be sure to use this sheet as your cover sheet.

1. Curve Fitting

Go to the class website and download the data set `ExamData1.mat`. To load the data into Matlab, type `load ExamData1` You'll see two arrays, `Year` and `Carb`. These give the atmospheric levels of Carbon Dioxide in parts per million for the years between 1764 and 1995. We want to try to model this data:

- Assume the model equation is: $y = Ae^{kt}$. Find the value of A, k that best fit this model, and plot the result (with the original data).
- Do you get a better line if you plot year versus $\log(\log(\text{Carb}))$? What is the model equation in this case? Plot the result that you get using this transformation of the data.
- In both cases, the fit was not perfect (be sure to look at the errors). Discuss what this means in terms of the rate of growth of the atmospheric carbon dioxide.

2. Population Modeling

We showed in class that for a model of predator-prey, we could write the population at time n as:

$$\begin{aligned} f_{n+1} &= (1 - k_1)f_n + k_3f_nr_n \\ r_{n+1} &= (1 + k_2)r_n - k_4f_nr_n \end{aligned}$$

Write some Matlab code that will simulate the fox and rabbit populations if:

$$k_1 = 0.01, k_2 = 0.0005, k_3 = 0.0001, k_4 = 0.0001$$

with the initial populations 25, 100 for foxes and rabbits, respectively. Go out to 6,001 time steps.

Turn in the plot that you get.

3. Consider the difference equation: $x_{n+2} + \frac{3}{4}x_{n+1} + \frac{1}{8}x_n = 0$.

- Give the closed form solution: (e.g., $x_n = ?$) if $x_0 = 0, x_1 = 1$. Be sure to show your (hand-done) algebra! For any initial condition, what will happen to x_n as $n \rightarrow \infty$?
- (For the next question, a sample Maple worksheet is on the class website, `SampleMaple.mws`).

Have Maple give you the solution for the following second order linear *nonhomogeneous* equation:

$$x_{n+2} + \frac{3}{4}x_{n+1} + \frac{1}{8}x_n = b_n$$

where (i) $b_n = \sin(n)$, and (ii) $b_n = 1 + n$ and use the initial conditions $x(0) = 1, x(1) = 1$. Compare these solutions to the solution we obtained for the nonhomogeneous first order equation, $x_{n+1} = \alpha x_n + b_n$ - specifically, is there a homogeneous part and a particular part?

Using this information, if $b_n = c_0 + c_1n + c_2n^2$, what form is the particular part of the solution?