Complex Numbers and Shear Transformations

Please turn in the solutions to this worksheet with your homework on Wednesday.

For questions 1 and 2, let:

\( z_1 = -1 + 3i, \ z_2 = 5i \ z_3 = 2 \ z_4 = -1 - i \)

1. Perform the given operation, and write the result as \( a + bi \).
   
   (a) \( z_1 + z_4 \)
   
   (b) \( \frac{z_1}{z_4} \)
   
   (c) \( z_1z_2, z_1z_4 \)
   
   (d) \( \text{Re}(z_1), \text{Re}(z_2), \text{Im}(z_2), \text{Im}(z_4) \)
   
   (e) \( \bar{z}_1, \bar{z}_3, \bar{z}_2 \)
   
   (f) \( |z_1|, |z_3| \)

2. (Optional) Write \( z_1, z_2, z_3, z_4 \) in polar form, \( re^{i\theta} \)

The next set of exercises asks you to write each matrix \( A \) as \( PSP^{-1} \), where \( P = [v, q] \), and \( D = \begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_1 \end{bmatrix} \), where \( q \) solves: \((A - \lambda I)q = v\).

\[
A = \begin{bmatrix} 1 & 9 \\ -1 & -5 \end{bmatrix}, \ A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, \ A = \frac{1}{2} \begin{bmatrix} -5 & 3 \\ -3 & 1 \end{bmatrix}
\]