Complex Numbers Homework

For questions 1 and 2, let:

$$z_1 = -1 + 3i, \;\; z_2 = 5i \;\; z_3 = 2 \;\; z_4 = -1 - i$$

1. Perform the given operation, and write the result as $a + bi$.

   (a) $z_1 + z_4$

   (b) $\frac{z_1}{z_4}$

   (c) $z_1 z_2$, $z_1 z_4$

   (d) $\mathrm{Re}(z_1), \mathrm{Re}(z_2), \mathrm{Im}(z_2), \mathrm{Im}(z_4)$

   (e) $\bar{z}_1, \bar{z}_3, \bar{z}_2$

   (f) $|z_1|$, $|z_3|$

2. Given the previous z_1, z_4, write the complex multiplication $z_1 z_4$ as matrix-vector multiplication in $\mathbb{R}^2$, and check your answer. Repeat the question with $z_4 z_1$ and $z_2 z_4$

3. Write the following in polar form, $re^{i\theta}$. If the argument is not a "nice" angle, use Matlab's `atan2`.

   (a) $z_1 = 1 + i$ (b) $z_2 = -0.5 - 0.4i$ (c) $z_3 = 3 - 2i$ (d) $z_4 = 3$ (e) $z_5 = -i$

4. Using the polar forms from the previous question, compute $z_1 z_5$ and $z_2 z_3$.

5. Download the "letter E" Matlab file from our class website. This is a short function that will generate points in the complex plane that forms a letter "E". To do this: Right click on the link, and save the file as "letterE.m" (without the quotes). In Matlab, type: X=letterE;

   To view the graph, type in: plot(X);

   Try performing some complex arithmetic to the points in X. For example,

   - What happens to the points if you multiply the vector by a complex number- For example, $\frac{1}{3} + \frac{1}{2}i$? or $1 - i$?
   - What happens to the points if they are transformed by some nonlinear function $F(z)$? For example, $F(z) = z^2$? or $F(z) = \sin(\frac{1}{5}z)$? [1] or $F(z) = \ln(z)$? You don't have to answer the question, but do print out the results-

6. We can define the natural logarithm of a complex number, $a + bi = re^{i\theta}$ by:

   $$\ln(a + bi) = \ln(r) + i\theta$$

   By definition, $\ln(0)$ is not defined. This is an extension of the usual logarithm, where $b = 0$ and $a > 0$. In fact, using this definition of the log, we can compute the logarithm of negative numbers (which is not defined over the real numbers).

[1] The $\sin(z)$ is defined much like the hyperbolic sine is defined, $\sin(z) = (e^{iz} - e^{-iz})/2i$

For example, $\ln(-1) = \ln(1) + i\pi = i\pi$. This also says that $e^{i\pi} = e^{\ln(-1)} = -1$.

Given the four complex numbers in question 3, compute the logarithm of each (Hint: You've already computed the polar forms!)