Review Material, Exam 3

The exam will cover the material we have discussed in class and studied in homework from Sections 4.4, 4.5, 4.6, 4.7, 5.1, 5.2, 5.3, 5.5 (Recall that the handout covered the computations of the eigenvalues and eigenvectors), and and 6.16.3.

Key Definitions

- Coordinates
- isomorphism
- dimension
- rank (of a matrix)
- A is similar to B
- characteristic equation
- Eigenvalue, eigenvector
- eigenspace
- diagonalizable matrix
- Dist between vectors
- the length of a vector
- Angles in $\mathbb{R}^n$.
- Orthogonal vectors
- orthogonal set
- orthogonal basis
- orthonormal basis
- projections
- orthogonal matrix
- Algebraic, Geometric multiplicity of an eigenvalue
- defective matrix

Key Theorems

Chapter 4:

Theorem 7, 8, 10 (makes the notion of dimension well-defined), 12 (How to compute a basis), 14 (Rank Theorem), 15 (Change of coordinates)

Chapter 5:

Theorems 1, 2, 5 (Diagonalization Theorem).

Chapter 6:

Theorem 2 (Pythagorean Theorem),
Theorem 3 (Null(A) is orthog to Row(A)),
Theorem 7 (Matrices with orthonormal columns are rigid rotations).
Theorem 8 (Orthogonal Decomposition Theorem).
Theorem 9 (Best Approximation Theorem),
Theorem 10 (Projection into a subspace W) generalizes Theorem 5 and gives the key quantity in Theorem 8.

Skills (partial list)

- Show that any finite (n) dimensional vector space is isomorphic to $\mathbb{R}^n$.
- Compute the coordinates of a vector with respect to a given basis, and with respect to an orthogonal basis.

- Find the change of coordinates matrix.
- Use the Rank Theorem to relate dimensions of subspaces (especially the 4 fundamental subspaces) and facts about matrix equations.
- Determine if a number (or vector) is an eigenvalue (or eigenvector) of a matrix.
- Find the characteristic equation and eigenvalues of a 2×2 matrix. Find the eigenvalues of a triangular matrix. Find a basis for an eigenspace. (NOTE: For matrices larger than 2×2, the eigenvalues would either be given, or the matrix would have a special form).
- If A is diagonalizable, find P and D such that $A = PDP^{-1}$. Show how to compute high powers of a diagonalizable matrix.
- Computations related to the geometry of $\mathbb{R}^n$: Compute length of vector, distance between vectors, angle between vectors. Check a set for orthogonality.
- Compute the orthogonal projection of a vector onto a vector, project a vector onto a subspace (in particular, a line or plane that includes the origin)
- Decompose a vector into a component in the direction of $\mathbf{u}$ and a component orthogonal to $\mathbf{u}$.
- Decompose a vector into the sum of a vector in W and a vector in $W^{\perp}$.
- Find the vector in a subspace W that is closest to a specified vector. Find the distance from a vector to W.