Homework: Complex Numbers

Read over Appendix B, page A3.

1. What is the **argument** of a complex number? Illustrate your answer with the number $-1+\sqrt{3}i$.

2. Perform the indicated operation, and write the result in standard form, $a+bi$:

   (a) $(1+i)(3-2i)$

   (b) $\dfrac{1}{i}-\dfrac{2}{1+i}$

   (c) $|3-2i|$

   (d) If $z=3+i$, compute $\overline{z}z$, and $1/z$.

3. If $A=\begin{bmatrix} 1+2i & 3-i \\ 3+i & 2-4i \end{bmatrix}$, show that the second row is $1-i$ times the first (and the matrix is therefore not invertible).

4. Points in the plane can be identified with complex numbers as: $(a,b)\leftrightarrow a+bi$. If we rotate the point $(1,0)$ by 45 degrees, we get $(0,1)$. Similarly, $(0,1)$ rotated by 45 degrees becomes $(-1,0)$. Using the identification, show that "Multiplication by i is rotation in the plane by 45 degrees", at least for the two points in question.

5. The remaining questions deal with **Euler's Formula**:

$$\mathrm{e}^{i\theta}=\cos(\theta)+i\sin(\theta)$$

   (a) Use Euler's Formula to obtain *the most beautiful theorem in mathematics* [1], Euler's Identity:

$$\mathrm{e}^{i\pi}+1=0$$

   (b) On page A6, we can replace the polar form: $z=|z|(\cos(\theta)+i\sin(\theta))$ by its more compact form, $z=|z|\mathrm{e}^{i\theta}$. Write the following complex numbers in this form:

   - $1+i$
   - $3i$
   - 4
   - $2-3i$ (use a calculator for θ).

   (c) Use your previous answer to write the product $(1+i)(2-3i)$ in polar form:

   (d) Write e^{1-2i} in the form $a+bi$.

[1] Mathematical Intelligencer poll