Take Home Portion: Final Exam for Math 300, Spring 2011

Instructions: You may work with one partner and you may use your textbook, class notes, and the material from our class website (write the name of the person you work with at the top the script file). Each person should email me the "published" script file for each question in PDF format. This portion of the exam is worth 30% of the overall final exam score.
Due date: Tuesday, May 17, 11PM.
NOTE: I can answer questions about the material we have looked at in the past, but not direct questions about the exam unless you're having technical difficulties downloading or something like that.

1. Download the file `FinalExamData1` from the class website. If you load it (be sure the command window is in the right directory, then type `load FinalExamData1`), two matrices, A and B will appear.

   (a) Verify that the Col(A) and Col(B) have the same dimension.

   (b) Determine whether or not Col(A) and Col(B) are the same subspace of $\mathbb{R}^5$. Explain what you calculated and why it worked.
   HINT: The QR decomposition for a general matrix A in Matlab is: `[Q,R]=qr(A,0)`

2. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}$$

   (a) Compute $\hat{\mathbf{b}}$ and $\mathbf{z}$ from the the orthogonal decomposition $\mathbf{b} = \hat{\mathbf{b}} + \mathbf{z}$, where $\hat{\mathbf{b}}$ is in the columnspace of A.

   (b) Verify (in Matlab) that $\mathbf{z} \in \text{Null}(A^T)$.

   (c) Use Matlab to obtain the least-squares solution to $A\mathbf{x} = \mathbf{b}$ by explicitly computing the normal equations. Call your answer $\hat{\mathbf{x}}$ or in Matlab `xhat`, and verify that $A\hat{\mathbf{x}} = \hat{\mathbf{b}}$

   (d) Compute the distance from $\mathbf{b}$ to Col(A).

3. Using 4 faces from class (any four you like), perform the Gram-Schmidt process on them. Plot the resulting figures together on a figure (see below)!

   Details: The file `FacesBW` that you'll download from the class website has a matrix H that is 1295504×15, corresponding to 15 column vectors (pick any 4 to work with).

   To visualize the vector as an array, each vector in $\mathbb{R}^{1295504}$ can be arranged into a 1204×1076 array, and that corresponds to an image. For example, to visualize the 8th column as a face, type:

```
Temp=reshape(H(:,8),1204,1076);
imagesc(Temp);
colormap(gray);
axis equal; axis off;
```

To see four faces in one plot (in this case, we'll plot faces 8, 10, 3, and 11):

```
Temp1=reshape(H(:,8),1204,1076);
Temp2=reshape(H(:,10),1204,1076);
Temp3=reshape(H(:,3),1204,1076);
Temp4=reshape(H(:,11),1204,1076);
subplot(2,2,1)
imagesc(Temp1);
colormap(gray);
axis equal; axis off;
subplot(2,2,2)
imagesc(Temp2);
axis equal; axis off;
subplot(2,2,3)
imagesc(Temp3);
axis equal; axis off;
subplot(2,2,4)
imagesc(Temp4);
axis equal; axis off;
```

For fun, you can put together a "movie" of all the faces:

```
figure
for j=1:15
    Temp=reshape(H(:,j),1204,1076);
    imagesc(Temp);
    axis equal; axis off;
    colormap(gray);
    M(j)=getframe;  %Grab the current image
    pause(0.3);  %Pause so you can see the figure
end
movie(M,4);  %Play the movie in M four times
```

For a strange look, leave off the colormap command... Groovy, baby!