Content for Exam 2, Linear Algebra

Exam 2 will cover material from Chapters 3 and 4, determinants and vector spaces. You will be allowed to use a calculator for arithmetic only. You may not use the text or notes. You will have approximately 50 minutes for the exam, and you may start at 12:50 PM.

Sections 3.1-3.3

1. Skills

   - Be able to compute determinants using a cofactor expansion along any row or column.
   - Compute a determinant for upper or lower triangular matrix.
   - Be able to compute a determinant by first performing row reduction.
   - Use Cramer's Rule to solve a system,
   - Compute the volume of a parallelepiped, area of a parallelogram.

2. Properties of the determinant. For the following, assume A, B are square matrices. For the last item, assume A is invertible.

   - A is invertible only if $\det(A) \neq 0$.
   - If A is $n \times n$, then $\det(kA) = k^n \det(A)$.
   - $\det(AB) = \det(A)\det(B)$
   - $\det(A^T) = \det(A)$
   - $\det(A^{-1}) = 1/\det(A)$

Vector Spaces, (4.1-4.6)

Here are some notes about Chapter 4

1. You don't need to memorize the 10 axioms on page 217.

2. Be familiar with some template vector spaces:

$$\mathbb{R}^n, \mathbb{P}_n, C[a,b], M_{m\times n}$$

3. Know these definitions: A linearly independent set, a subspace, a basis, the coordinates of $\mathbf{x}$ (with respect to a given basis), the dimension of a subspace, an isomorphism, the rank of a matrix. The four fundamental subspaces associated with a matrix A (be able to define each one), the kernel of a transformation, the change of coordinates matrix,

4. Theorems for computation (These are theorems you do not need to state, but can use to assist in computations):

   Chapter 3: 1, 2, 3

   Chapter 4: 4, 5, 6, 7, 9, 11, 12, 13,

5. Theorems to know:

   Chapter 3: 4, 5, 6, 7, 9

   Chapter 4: 1, 2, 3, 8, 10, 14

   (Be able to prove 2 and 3 from Ch. 4).

6. Skills:

   - Prove that a given set is or is not a subspace.
   - Given a matrix A, be able to compute a basis for the column space, the null space and the row space (not the null space of A^T).
   - Find the kernel of a given transformation and describe the range of the transformation.
   - Understand how row operations affect the the 4 fundamental subspaces (for example, the subspaces for a matrix A versus its RREF B).
     - Row operations do not affect the relationship among the columns of A, but they do affect the column spaces (the column spaces of A, B may not be the same).
     - Row operations do affect the relationship among the rows of A, but the row spaces of A, B are the same.
     - Row operations do not affect the set of solutions to $A\mathbf{x} = \mathbf{0}$, so the null spaces of A, B are the same.
   - Find the coordinates of a vector given a basis (both in $\mathbb{R}^n$ using the change of coordinates matrix, and for vector spaces that are not $\mathbb{R}^n$, like $\mathbb{P}_n$.
   - Be able to compute the dimension of a vector space.
   - Be able to compute the rank of a matrix. Use that to compute the dimensions of the four fundamental subspaces.