Review Material, Exam 3

The exam will cover the material we have discussed in class and studied in homework from Sections 4.9, 5.1-5.6 (exc 5.4), 6.1-6.2. A calculator that does not perform symbolic computations may be used on the exam (you just need a basic scientific calculator).

Key Definitions

- Stochastic Matrix
- Markov Chain
- Equilibrium vector
- A is similar to B
- Char. polynomial
- Char. equation
- Eigenvalue, eigenvector
- Eigenspace
- Diagonalizable matrix
- Dist between vectors
- Length of a vector
- Angle between vectors
- Orthogonal vectors
- Orthogonal set
- Orthogonal basis
- Orthogonal matrix
- Orthonormal basis
- Orthogonal complement
- Orthogonal Projection
- Orthogonal matrix
- Algebraic, Geometric multiplicity of an eigenvalue
- Defective matrix
- Conjugate, argument, magnitude
- Euler's Formula

For eigenvalues, recall the properties of the determinant as well.

Key Theorems

Chapter 4:

Theorem 18 (Convergence of Markov Chain)

Chapter 5:

Theorem 1 (Evals of triangular matrix), Theorem 2 (Lin Ind Evecs), Be able to prove Theorems 4 and 5. Be able to use Theorems 7 and 9.
Relationship of the eigenvalues to the invertibility of a matrix.

Chapter 6:

Theorem 2 (Pythagorean Theorem), Theorem 3 (Null(A) is orthog to Row(A)),

Skills (partial list)

- Be able to compute basic operations using complex numbers (in particular, multiplication and division). Be able to convert between complex numbers and the polar form using Euler's Formula (not in text). Use the handout from class as your guide.

- Be able to form a Markov Chain, given some physical situation. Be able to determine equilibria, and more generally (not necessarily with stochastic matrices), determine the long term behavior of $\mathbf{x}_{n+1} = A\mathbf{x}_n$
- Diagonalize a matrix, if possible.
- Factor a matrix if it has complex eigenvalues (Theorem 9, p. 340)
- Determine if a number (or vector) is an eigenvalue (or eigenvector) of a matrix.
- Find the characteristic equation and eigenvalues of a 2×2 matrix. Find the eigenvalues of a triangular matrix. Find a basis for an eigenspace. (NOTE: For matrices larger than 2×2, the eigenvalues would either be given, or the matrix would have a special form).
- If A is diagonalizable, find P and D such that $A = PDP^{-1}$. Show how to compute high powers of a diagonalizable matrix.
- If $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, explain the "action" of A using scaling and/or rotations.
- If A is 2×2 with complex eigenvalues, find P, C so that $A = PCP^{-1}$.
- Given A with larger dimensions, be able to factor it using a combination of techniques from the previous two items ($A = PDP^{-1}$ and $A = PCP^{-1}$).
- Computations related to the geometry of $\mathbb{R}^n$: Compute length of vector, distance between vectors, angle between vectors. Check a set for orthogonality. Normalize a vector.
- Compute the orthogonal projection of a vector onto a vector, project a vector onto a subspace (in particular, a line or plane that includes the origin)
- Decompose a vector into a component in the direction of $\mathbf{u}$ and a component orthogonal to $\mathbf{u}$.