Homework for 6.3

22. (True or False)

    (a) True- If a vector is orthogonal to itself, then $\mathbf{x} \cdot \mathbf{x} = 0$, and by the properties of the dot product, $\mathbf{x} = 0$. Therefore, the zero vector is the only vector that is included in both a subspace and its perpendicular complement.

    (b) True- The projection of a vector into a subspace that is represented by a basis of orthogonal vectors is the sum of the projections to each basis vector.

    (c) False by the Orthogonal Decomposition Theorem; $\mathbf{y}$ is best approximated by $\text{Proj}_W(\mathbf{y})$

    (d) False: $UU^T\mathbf{x}$ is the projection of $\mathbf{x}$ onto the column space of U (so true if $\mathbf{x}$ is in the column space).

    (e)

    Matlab for # 25, # 26 (template given), but the ideas are:

25 The matrix U has orthogonal columns, which was the purpose of computing U^TU. The projection to the column space of U is: `yhat=U*U'*y`, so the distance (not necessary, but nice) to the subspace is: `norm(y-yhat)`, which was approximately 1.26 units.

26 Same idea, same matrix. In this case, the distance was `norm(b-U*U'*b)`, or about 2.12 units.