Review Questions (Last part of class)
Math 338

Important Notes:

- Things you will be provided, if necessary: Sheet of distributions (the latest one, also linked from our regular class website), and any tables you might need.
- Bring a calculator. You may write anything you like on both sides of a 3 inch by 5 inch card and bring it with you.

This part of the review focuses primarily on the last part of the class. You should also review the old exams and old review sets, quizzes and homework.

1. Define a **random sample:**
2. Define a **statistic:**
3. Define the sample mean and sample variance.
4. What is the difference between $\bar{X}, S^2$, $\bar{x}, s^2$, and μ, σ^2?
5. When is $\hat{\Theta}$ an *unbiased* estimator of θ?
6. In class, we constructed an estimate for an interval in which the true population mean μ could be found with probability $\hat{P}$. What was that interval, if we have to use s rather than σ?

   Hint: Think about $P(|T| < t_{\alpha,\nu}) = \hat{P}$
7. When is the $t-$distribution used in association with sampling? When is the χ^2 distribution used? (In particular, pay attention to any conditions on their use)
8. If X_1 is χ^2 with 1 degree of freedom, and X_2 is χ^2 with 3 degrees of freedom, then what distribution is $Y = X_1 + X_2$ (assume independence of X_1, X_2). Show your work, do not simply state the answer.
9. The four principal game fish in Clear Lake are bluegills, crappie, small-mouth bass and large-mouth bass. The weights of these, amazingly, has a χ^2 distribution with the following parameters:

Name	deg. of freedom
Bluegills	8 oz
Crappie	13 oz
Small MB	1 lb 3 oz
Large MB	3 lb 2 oz

   where one lb (pound) is 16 oz (ounces). Assuming that the numbers of these fishes are independent, if Fisherperson Terry catches 6 bluegills, 5 crappie, 3 small mouth bass, 3 large mouth bass, what is the probability that the total weight of these 17 fish is over 19 pounds?

10. Given a random sample of size n from a population with *known* mean μ, show that

$$\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2$$

is an unbiased estimator of the population variance σ^2.

11. Show that the following is a biased estimator of the binomial parameter θ.

$$\frac{X+1}{n+2}$$

12. The claim that the variance of a normal population is 4 will be rejected if the variance of a random sample of size 9 exceeds 7.7535. What is the probability that the claim will be rejected (even though the actual parameter is $\sigma^2 = 4$)? That is, check the probability that S^2 will be greater than 7.7535.

13. A random sample of size 100 is taken from an infinite population with the mean $\mu = 75$ and $\sigma^2 = 256$. Compare the probabilities we get that $\bar{X}$ will fall between 67 and 83 using (i) Chebyshev's Inequality, and (ii) CLT.

14. Given an iid random sample $X_1, X_2, \ldots, X_n$, we said that the random variable $\bar{X}$ has its own distribution- What is its mean and variance? Is there a relationship between that and the sample mean and sample variance (biased or unbiased estimators perhaps?)

15. The width of a fence board that is marked 6 inches will actually have a mean width measurement of $\mu = 5.5$ inches (that is true) with a standard deviation of 0.24 inches (that is made up). What is the probability (using the CLT) that the total length of 100 boards placed side by side (with the gap between negligible) will be between 546 and 554 inches?

16. In a study of television viewing habits, it is desired to estimate the number of hours that teenagers spending watching per week. If it is reasonable to assume that $\sigma = 3.2$ hours, how large a sample is needed so that it will be possible to assert with 95% probability that $|\bar{X} - \mu|$ is less than 20 minutes? (NOTE 1: Use hours for units. NOTE 2: Assume that the random sample is from a normal distribution)

17. Given a random sample of size n from a Γ distribution, find formulas for α, β in terms of the first two moments.

18. Given a random sample of size n from a geometric population, find formulas for estimating its parameter θ by (a) Method of moments, (b) MLE (Maximum Likelihood Estimation)

19. If X and Y are two independent rvs having identical gamma distributions, find the joint pdf of the random variables $U = \frac{X}{X+Y}$ and $V = X + Y$.

20. Given 8 data values, and

$$\sum_{i=1}^{8} x_i = 108 \qquad \sum_{i=1}^{8} x_i^2 = 1486$$

Compute the sample mean and sample variance.

If the population was a Γ

21. Prove the following, using the MGF technique: If X_1, X_2 are independent rvs, and X_1 has a χ^2 distribution with dof ν_1, and $X_1 + X_2$ has χ^2 with dof $\nu > \nu_1$, then X_2 is χ^2 with dof $\nu - \nu_1$.

22. Consider two random variables X and Y whose joint pdf is given by: $f(x, y) = \frac{1}{2}$ if $x > 0, y > 0$ and $x + y < 2$ (zero elsewhere). Find the pdf of $U = Y - X$.

23. Let X and Y be iid exponential with $\theta = 1$. Let $Z = \frac{1}{2}(X + Y)$. Find the pdf of Z by the CDF technique.

24. Given a random sample of size n from a Gamma distribution with known parameter $\alpha = 2$, find the MLE of the parameter β.