

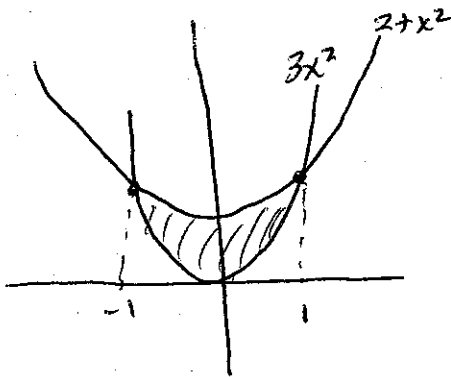
Extra Practice Problems, Double Integrals

Given below is an area D in the plane. Set up the integral

$$\iint_D f(x, y) dA$$

first in the most natural way, then change the order of integration (if possible). Need more information? Look at Section 15.3 of Stewart's Calculus, for example.

1. D is the region bounded below $y = 3x^2$ and above $y = 2 + x^2$.

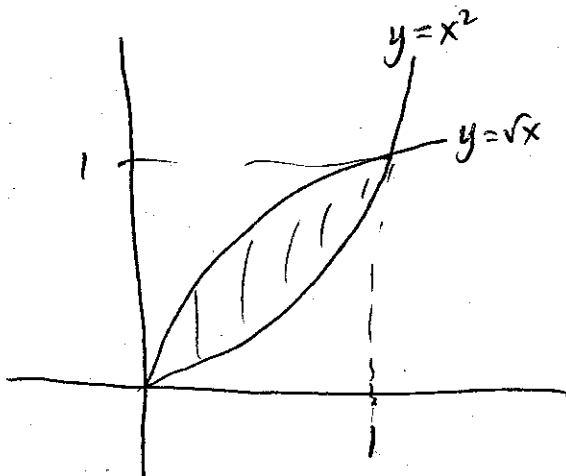


(a) y first: $3x^2 \leq y \leq 2 + x^2$
 $-1 \leq x \leq 1$

(b) Reverse: Not very feasible;
 we run into trouble if
 $2 < y < 3$, especially.



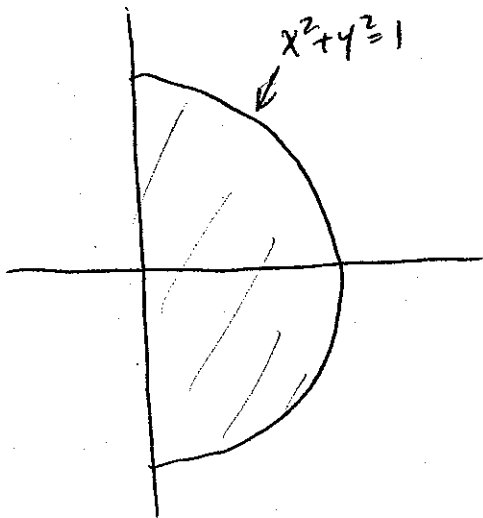
2. D is the area bounded between $y = \sqrt{x}$ and $y = x^2$ and $0 \leq y \leq 1$.



(a) y first: $x^2 \leq y \leq \sqrt{x}$
 $0 \leq x \leq 1$

(b) x first:
 $y^2 \leq x \leq \sqrt{y}$
 $0 \leq y \leq 1$

3. D is the right half of the unit circle (in Quadrants I and IV).



(a) y first:

$$-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$$

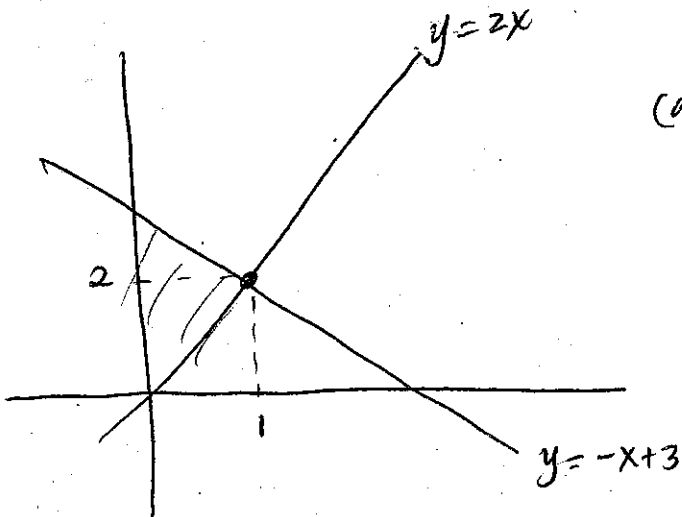
$$0 \leq x \leq 1$$

(b) x first:

$$0 \leq x \leq \sqrt{1-y^2}$$

$$-1 \leq y \leq 1$$

4. D is the area bounded by $x \geq 0$, $y \geq 0$, $x + y \leq 3$, and $y \geq 2x$.



(a) y first:

$$2x \leq y \leq 3-x$$

$$0 \leq x \leq 1$$

(b) x first: Break into 2:

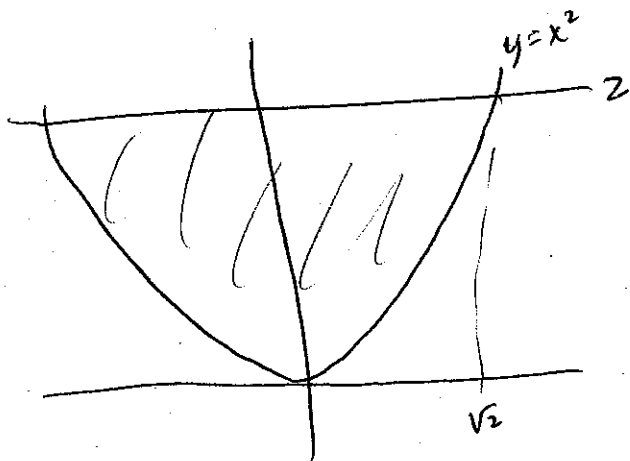
$$0 \leq x \leq \frac{1}{2}y$$

$$0 \leq y \leq 2, \text{ then}$$

$$0 \leq x \leq 3-y$$

$$2 \leq y \leq 3$$

5. D is the area under the line $y = 2$ and above the parabola $y = x^2$.



(a) y first:

$$x^2 \leq y \leq 2$$
$$-\sqrt{2} \leq x \leq \sqrt{2}$$

(b) x first:

$$-\sqrt{y} \leq x \leq \sqrt{y}$$
$$0 \leq y \leq 2$$