Illustrative Example: Suppose a coin is biased so that it comes up heads 75% of the time. Let the rv X_i be Bernoulli, so that $X_i = 1$ if the coin comes up heads, $X_i = 0$ otherwise. The experiment will be to toss a coin three times, and compute the distribution of the sample mean, $\bar{X}$.

First, fill in the table:

OUTCOME	PROBABILITY	$\bar{X}$
HHH	$\left(\frac{3}{4}\right)^3 = \frac{27}{64}$	$\frac{1+1+1}{3} = 1$
HHT		
HTH		
HTT		
THH		
THT		
TTH		
TTT		

Now compute the pdf of $\bar{x}$:

And the mean of $\bar{x}$ is? The standard deviation is? Compare these to Theorem 8.1.

Extra Practice Problems- Central Limit Theorem

A runner attempts to pace off 100 meters for an informal race. Assume that her paces are independently distributed with mean $\mu = 0.99$ meters and standard deviation $\sigma = 0.1$ meters. Use the Central Limit Theorem to find (approximately) the probability that the total length of the 100 paces will differ from 100 meters by no more than 2 meters (above or below).

Suppose that people at a Whitman party randomly pour drinks from a bottle containing 67.5 ounces of a certain liquid (lemonade?). Suppose also that the expected weight of each drink is 2 ounces, and the standard deviation is one half ounce, and all drinks are poured independently. Use the Central Limit Theorem to determine (approximately) the probability that the bottle will not be empty after pouring 36 drinks.