

Random Sampling of Questions

1. The probability that rain is followed by rain is 0.8, a sunny day is followed by rain is 0.6. Find the probability that one has two rainy days then two sunny days.

2. Let:

$$f(x) = k|x - 2|, \quad \text{for } x = -1, 0, 1, 3$$

- Find k so that f is a PDF.
- Find the expected value of X .
- Find the expected value of X^2 .

3. A balanced die is tossed twice. Let A be the event that an even number comes up on the first toss. Let B be the event that an even number comes up on the second. Let C be the event that the first two tosses gave the same number. Are A, B, C

- pairwise independent
- independent?

4. State Bayes' Theorem.

5. If A, B are independent prove that A' and B' are independent.

6. What is the difference between saying that A, B, C are independent events, versus saying that X, Y, Z are independent random variables?

7. In a poker game, 5 cards are dealt from a deck of 52. What is the probability of getting a four-of-a-kind?

8. If the joint PDF is given by:

$$f(x, y, z) = \frac{xyz}{108} \text{ for } x = 1, 2, 3; y = 1, 2, 3; z = 1, 2$$

- (a) Find the joint marginal distribution of X and Y .
- (b) Find the conditional distribution of Z given $X = 1$ and $Y = 2$.
- (c) Find the marginal distribution of X .
- (d) Are X, Y, Z independent random variables? Prove or Disprove.

9. How many ways can a college team playing 10 games end up with 5 wins, 4 losses and a tie?

10. A carton contains 15 light bulbs of which one is defective. In how many ways can you choose 3 of the bulbs and:

- (a) Get the one that is defective?
- (b) Not get the one that is defective?
- (c) Find the probability of choosing the defective bulb.

11. Given that 8% of the population has diabetes, a health department comes in and gives tests. It correctly diagnoses 95% of all persons with diabetes as having the disease, and incorrectly diagnoses 2% of all persons without diabetes as having the disease.

- (a) Find the probability that the health department will diagnose someone in the population as having the disease.

- (b) Find the probability that someone diagnosed by the health department as having the disease actually has it.
12. Let $f(x) = \frac{1}{x \ln(3)}$ for $0 < x < 3$ (zero elsewhere). Find $E(X)$ and $E(X^2)$, then use the results to find $E(2X^2 - X + 1)$
13. Experiment: Roll three standard dice. Observe the outcomes.
- How many ways can the three dice all come up with the same number of points?
 - How many ways can two of the three come up with the same number, but the third die is different?
 - How many ways can all three dice come up with different numbers?
14. Verify that $f(x) = \frac{2x}{k(k+1)}$ for $x = 1, 2, 3, \dots, k$ can serve as the probability distribution of a random variable with the given range.
15. State the Law of the Unconscious Statistician.
16. We have two men and four women making up a committee of three.
- What is the probability of choosing no men? one man? two men?
 - Can you come up with a probability distribution function (PDF) for this using binomials?
17. How many ways can a bakery distribute its 7 unsold apple pies to 4 food banks? How does this change if every food bank must get at least one?
18. If I have four skirts, seven blouses and three sweaters, how many ways can I pick 2 skirts, three blouses and one sweater to take along on a trip?
19. Two cards are randomly drawn from a deck of 52. Find the probability that both cards will be greater than 3 and less than 8.
20. If the joint density of X, Y is given by

$$f(x, y) = \begin{cases} 4xy & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

- Compute $f(x|y)$ and $f(y|x)$.
- Set up an integral to determine $P(X + Y < 1)$
- Find $F(x, y)$. Be careful of the regions (like we did in class).
- Are X, Y independent random variables?
- Set up an integral to determine $E(X)$ and $E(Y)$. Is the product the same as $E(XY)$? Do you think that always happens?