Misc. Review Problems, Exam 2
Math 338
Fall 2007

As usual, these questions are here to give you a chance to review problems out of context, and not necessarily exhaustive. You should be sure that you understand the quizzes and homework problems that have been assigned. You will be given a list of the distributions that we have looked at- Note that some were missing from Ch 5, all from Ch 6 will be included.
Specific Sections:
4.3-4.8, 5.1-5.7, 6.1-6.6 (omit 6.4)

1. Draw a flow chart that relates all the different distributions (if such a connection exists). For example, the geometric distribution is a special case of ??? (using what parameters)?

2. Exercise 5.19: Compute the mean and variance of the negative binomial distribution (the technique is what is important here- factor out what we need and manipulate the sum).

3. The distribution of some weeds across my yard looks like it might be random. Suppose there are 75 dandelions in my rectangular yard, and I divide my yard into a 10×10 grid.

   The following is a summary of what I got- Is the distribution random?

Number of Weeds	0	1	2	34	
Number of squares	47	36	13	3	1

   By the way, if you x is the first row and y is the second, what should you get with $\sum x_i y_i$? With $\sum y_i$?

4. Suppose that experience has shown that the length of time X required to conduct a periodic maintenance check on an ipod follows a Gamma distribution with $\alpha = 3.1$ and $\beta = 2$. A new maintenance worker takes 22.5 minutes to check the machine.

   Use Chebyshev's inequality to see if this length of time agrees with our past experience.

5. Find the moment generating function for the exponential distribution. Use it to verify the mean and variance.

6. Given that the moment generating function of the normal distribution is:

   $$M(t) = \mathrm{e}^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

   if X is a normally distributed rv with mean 4 and standard deviation 2, what is the moment generating function for $Y = -3X + 4$? What is the distribution of Y (i.e., can you recognize the mgf?)

7. A company manufactures and bottles juice in 16 oz containers. The amount actually dispensed has been observed to be approximately normal with mean 16 and standard deviation 1 oz. What proportion of bottles will have more than 17 oz dispensed into them?

8. Let $f(x;\theta)$ be the Bernoulli distribution. Find the mean and variance directly.

   Let X_i each be a Bernoulli rv with the same parameter θ, and they are independent. If $Y = X_1 + X_2 + X_3 + \cdots + X_n$, compute the mean and variance of Y.

9. Suppose a large stockpile of used pumps contains 20% that are in need of repair. A maintenance worker is sent to the stockpile with three repair kits. He selects pumps at random and tests them one at a time. If the pump works, he sets it aside for future work. If not, he repairs it. Suppose that it takes 10 minutes to test a pump that is in working order, and 30 minutes to test and repair a pump that does not work. Find the mean and variance of the total time it takes the maintenance worker to use the three repair kits.

10. Suppose X has a normal distribution, $n(x;2,3)$. Write $P(X \leq 4)$ so that you can look up the needed probabilities in Table III.

11. Find the moment generating function for the rv X, if X has the continuous pdf $f(x) = 1$ for $0 < x < 1$, zero elsewhere.

    Use it to find the mean and variance.

12. Using the definition of Γ, compute $Gamma(5)$. Hint: You might want to use a table for integration by parts, and the Lemma we had in class about polynomials and exponentials.

13. We said that we could use the Poisson distribution to approximate the Binomial distribution. Under what circumstances, and exactly how do we do it (given the parameters for the Binomial, what are the parameters for the Poisson)?

    We said that we could use the normal distribution to approximate the Binomial distribution. Under what circumstances, and exactly how do we do it (given the parameters for the Binomial, what are the parameters for the normal)?

14. Use the Maclaurin series for the mgf $M(t)$ in order to find the 4^{th} moment about the mean, if

    $$M(t) = \frac{1}{1-t^2}$$

    (Hint: Think Geo Series)

15. If $Y = aX + b$, and $M(t)$ is the mgf of X, find the mgf of Y:

16. Recall the series expansion of the exponential function, and use it to compute the expected value of $X(X-1)$.

17. Fill in the question marks: If the mgf is:

$$M(t) = e^{3t+8t^2}$$

then the random variable was a normal distribution with mean ??? and standard deviation ???

Given your answer, show what you get for the mgf of Z (and check the result of the previous exercise) if you let $Z = \frac{X-\mu}{\sigma}$

18. Let X be a normally distributed random variable with $\mu = 124$ and $\sigma = 7.5$.

Use Chebyshev's Inequality to estimate the probability that $64 < X < 184$.

Use Table III to get the probability that $64 < X < 184$.

19. A company employs n people. Its income is a Gamma distribution, $g(x; 80\sqrt{n}, 2)$ Its cost is $8n$. Find the number of people that will give the maximum expected profit (Income−Cost).

20. If X is a normal rv, $n(x; 2, 3)$, then use Table III to give:

- $P(X \geq 4)$
- $P(X \geq 1)$
- $P(-1 < X < 4)$

21. Compute the covariance of $f(x, y) = \frac{1}{4}(2x + y)$, $0 < x < 1$, $0 < y < 2$ (zero elsewhere).

22. Given a standard normal distribution, find the Maclaurin series for its moment generating function. Use it to find formulas for the third, fourth and sixth moments about the origin.

23. Show that the exponential distribution is memoryless. That is, that

$$P(X \geq t + T | X \geq T) = P(X \geq T)$$

24. Show that the geometric distribution is memoryless. That is,

$$P(X \geq t + T | X \geq T) = P(X \geq T)$$

25. Let X_1 be a uniform distribution on the interval $[1, 5]$, let X_2 be a normal distribution, $n(x; 3, 2)$ and let X_3 be another normal distribution, $n(x; -1, 3)$. Find the mean and variance of $Y = 2X_1 - 3x_2 + X_3$, assuming that X_1, X_2, X_3 are independent.

26. Find the moment generating function for the geometric distribution, and use it to find the first two moments about the origin.