Review Questions
Exam 3
Math 338

Important Notes:

- Things you will be provided, if necessary: Sheet of distributions and Moment Generating Functions (Note χ^2). Tables for the normal distribution, χ^2 distribution, $t-$distribution.
- You should bring a calculator with you- there will be some arithmetic to do. Calculators that do symbolic manipulation (like factorization or differentiation) are not allowed.
- You may write anything you like on both sides of a 3 inch by 5 inch card (or a 3×5 sheet of paper) and bring it with you.

As usual, you shouldn't take the review to be exhaustive- You should be sure that you understand the homework and quizzes, for example. You should go through the material yourself and use this as a self-test.

1. Define a **random sample:**

2. Why is the $t-$distribution used in association with sampling? Why is the χ^2 distribution used? (In particular, pay attention to any conditions on their use)

3. The four principal game fish in Clear Lake are bluegills, crappie, small-mouth bass and large-mouth bass. The weights of these, amazingly, has a Chi-square distribution with the following parameters:

Name	deg. of freedom
Bluegills	8 oz
Crappie	13 oz
Small MB	1 lb 3 oz
Large MB	3 lb 2 oz

   where one pound is 16 oz. Assuming that the numbers of these fishes are independent, if Fisherperson Terry catches 6 bluegills, 5 crappie, 3 small mouth bass, 3 large mouth bass, what is the probability that the total weight of these 17 fish is over 19 pounds?

4. If X_1, X_2, X_3, X_4, X_5 are iid with standard normal distributions, find c so that the random variable:

$$\frac{c(X_1 + X_2)}{\sqrt{X_3^2 + X_4^2 + X_5^2}}$$

   has a t distribution.

5. Given a random sample of size n from a Gamma distribution with known parameter $\alpha = 2$, find the MLE[1] of the parameter β.

6. The claim that the variance of a normal population is 4 will be rejected if the variance of a random sample of size 9 exceeds 7.7535. What is the probability that the claim will be rejected (even though the actual parameter is $\sigma^2 = 4$)?

7. A random sample of size 100 is taken from an infinite population with the mean $\mu = 75$ and $\sigma^2 = 256$. Compare the probabilities we get that $\bar{X}$ will fall between 67 and 83 using (i) Chebyshev's Inequality, and (ii) CLT.

8. Given an iid random sample $X_1, X_2, \ldots, X_n$, we said that the random variable $\bar{X}$ has its own distribution- What is its mean and variance? Is there a relationship between that and the sample mean and sample variance (biased or unbiased estimators perhaps?)

9. The width of a fence board that is marked 6 inches will actually have a mean width measurement of $\mu = 5.5$ inches (that is true) with a standard deviation of 0.24 inches (that is made up). What is the probability (using the CLT) that the total length of 100 boards placed side by side (with the gap between negligible) will be between 546 and 554 inches?

10. In a study of television viewing habits, it is desired to estimate the number of hours that teenagers spending watching pe week. If it is reasonable to assume that $\sigma = 3.2$ hours, how large a sample is needed so that it will be possible to assert with 95% confidence that the mean is off by less than 20 minutes?

11. Let Y be a random variable with pdf $f(y)$. Let $Z = G(Y)$. What should G be in order for Z to be uniform on $(0, 1)$?

12. Given a random sample of size n from a geometric population, find formulas for estimating its parameter θ by (a) Method of moments, (b) MLE (Maximum Likelihood Estimation)

13. If X and Y are two independent rvs having identical gamma distributions, find the joint pdf of the random variables $U = \frac{X}{X+Y}$ and $V = X + Y$.

14. The standard error is either $\sigma/\sqrt{n}$ or $s/\sqrt{n}$. What is the length of a confidence interval (using σ, then using s)?

    How would n have to change in order to halve a confidence interval?

[1] Maximum Likelihood Estimator

15. Find $z_{\alpha/2}$ if we are to construct an 85% confidence interval (two sided as is our usual practice):

16. Given 8 data values, and

$$\sum x_i = 108 \qquad \sum x_i^2 = 1486$$

Compute the sample mean and sample variance.

17. Prove the following, using the MGF technique: If X_1, X_2 are independent rvs, and X_1 has a χ^2 distribution with dof ν_1, and $X_1 + X_2$ has χ^2 with dof $\nu > \nu_1$, then X_2 is χ^2 with dof $\nu - \nu_1$.

18. Consider two random variables X and Y whose joint pdf is given by: $f(x,y) = \frac{1}{2}$ if $x > 0, y > 0$ and $x + y < 2$ (zero elsewhere). Find the pdf of $U = Y - X$.

19. Let X and Y be iid exponential with $\theta = 1$. Let $Z = \frac{1}{2}(X + Y)$. Find the pdf of Z by the CDF technique.

20. Show that the following is an unbiased estimator of σ^2 (which is the shared variance of X_1, X_2):

$$S_p^2 = \frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}$$