Homework, 7.4

1. Given the optimal solution below, find Δ so that the current basis remains optimal, if we want to change c_{22} from 12 to $12 + \Delta$.

   SOLUTION: Make the changes- Since c_{22} is not basic, the change is localized since the u's and v's don't change. The only other change is that the new value in the parentheses needs to be non-negative.

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
	8	6	10	9	
$u_1 = 0$		10	25		35
	9	$12 + \Delta$	13	7	
$u_2 = 3$	45	$(3 + \Delta)$	5		50
	14	9	16	5	
$u_3 = 3$		10		30	40
Demand	45	20	30	30	125

   From this we see that $\Delta > -3$.

2. Given the optimal solution below, find Δ so that the current basis remains optimal, if we want to change c_{32} from 9 to $9 + \Delta$. An extra table below is included, if you want to use it for your computations.

   SOLUTION: Since the $(3, 2)$ cell is basic, the u's and v's will need to be recomputed, which also means some of the the NBV cells also need recomputing.

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2 - \Delta$	Supply
	8	6	10	9	
$u_1 = 0$	(2)	10	25	$(7 + \Delta)$	35
	9	12	13	7	
$u_2 = 3$	45	(3)	5	$(5 + \Delta)$	50
	14	$9 + \Delta$	16	5	
$u_3 = 3 + \Delta$	$(5 - \Delta)$	10	$(3 - \Delta)$	30	40
Demand	45	20	30	30	125

   All together, we see $-5 < \Delta < 3$.

3. Given the optimal solution below, find the new optimal solution if we add Δ to Demand 2, Supply 3.

   SOLUTION: Making this change just means that the value in cell $(2, 3)$ (which is basic), just increases by Δ. The basis stays optimal as long as the value in the cell stays non-negative, so $\Delta > -10$.

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	$\boxed{8}$	$\boxed{6}$ 10	$\boxed{10}$ 25	$\boxed{9}$	35
$u_2 = 3$	$\boxed{9}$ 45	$\boxed{12}$	$\boxed{13}$ 5	$\boxed{7}$	50
$u_3 = 3$	$\boxed{14}$	$\boxed{9}$ 10+Δ	$\boxed{16}$	$\boxed{5}$ 30	40+Δ
Demand	45	20+Δ	30	30	125

4. Given the optimal solution below, find the new optimal solution if we add Δ to Demand 4, Supply 2. Also compute the change in z. An extra table is below if you want to use it.

   SOLUTION: By increasing cell $(2, 4)$ from zero to Δ, we create a loop. We then need to incorporate Δ into the existing basic solution (See the loop after the table).

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	$\boxed{8}$	$\boxed{6}$ 10+Δ	$\boxed{10}$ 25$-\Delta$	$\boxed{9}$	35
$u_2 = 3$	$\boxed{9}$ 45	$\boxed{12}$	$\boxed{13}$ 5+Δ	$\boxed{7}$	50+Δ
$u_3 = 3$	$\boxed{14}$	$\boxed{9}$ 10$-\Delta$	$\boxed{16}$	$\boxed{5}$ 30+Δ	40
Demand	45	20	30	30+Δ	125

Loop and result:

$$\begin{array}{c|c|c} 10+\Delta & 25-\Delta & \\ \hline & 5+\Delta & \Delta-\Delta \\ \hline 10-\Delta & & 30+\Delta \end{array} \qquad \Rightarrow \qquad \Delta > 10$$