We may find the new values of the decision variables as follows:

1 If x_{ij} is a basic variable in the optimal solution, then increase x_{ij} by Δ.

2 If x_{ij} is a nonbasic variable in the optimal solution, find the loop involving x_{ij} and some of the basic variables. Find an odd cell in the loop that is in row i. Increase the value of this odd cell by Δ and go around the loop, alternately increasing and then decreasing current basic variables in the loop by Δ.

REVIEW PROBLEMS

Group A

1 Televco produces TV picture tubes at three plants. Plant 1 can produce 50 tubes per week; plant 2, 100 tubes per week; and plant 3, 50 tubes per week. Tubes are shipped to three customers. The profit earned per tube depends on the site where the tube was produced and on the customer who purchases the tube (see Table 64). Customer 1 is willing to purchase as many as 80 tubes per week; customer 2, as many as 90; and customer 3, as many as 100. Televco wants to find a shipping and production plan that will maximize profits.

a Formulate a balanced transportation problem that can be used to maximize Televco's profits.

b Use the northwest corner method to find a bfs to the problem.

c Use the transportation simplex to find an optimal solution to the problem.

2 Five workers are available to perform four jobs. The time it takes each worker to perform each job is given in Table 65. The goal is to assign workers to jobs so as to minimize the total time required to perform the four jobs. Use the Hungarian method to solve the problem.

3 A company must meet the following demands for a product: January, 30 units; February, 30 units; March, 20 units. Demand may be backlogged at a cost of \$5/unit/month. All demand must be met by the end of March. Thus, if 1 unit of January demand is met during March, a backlogging cost of 5(2) = \$10 is incurred. Monthly production capacity and unit production cost during each month are given in Table 66. A holding cost of \$20/unit is assessed on the inventory at the end of each month.

a Formulate a balanced transportation problem that could be used to determine how to minimize the total cost (including backlogging, holding, and production costs) of meeting demand.

b Use Vogel's method to find a basic feasible solution.

c Use the transportation simplex to determine how to meet each month's demand. Make sure to give an interpretation of your optimal solution (for example, 20 units of month 2 demand is met from month 1 production).

4 Appletree Cleaning has five maids. To complete cleaning my house, they must vacuum, clean the kitchen, clean the bathroom, and do general straightening up. The time it takes each maid to do each job is shown in Table 67. Each maid

TABLE 64

	To (\$)		
From	Customer 1	Customer 2	Customer 3
Plant 1	75	60	69
Plant 2	79	73	68
Plant 3	85	76	70

TABLE 65

	Time (Hours)			
Worker	Job 1	Job 2	Job 3	Job 4
1	10	15	10	15
2	12	8	20	16
3	12	9	12	18
4	6	12	15	18
5	16	12	8	12

TABLE 66

Month	Production Capacity	Unit Production Cost
January	35	\$400
February	30	\$420
March	35	\$410

TABLE 67

	Time (Hours)			
Maid	Vacuum	Clean Kitchen	Clean Bathroom	Straighten Up
1	6	5	2	1
2	9	8	7	3
3	8	5	9	4
4	7	7	8	3
5	5	5	6	4

is assigned one job. Use the Hungarian method to determine assignments that minimize the total number of maid-hours needed to clean my house.

5[†] Currently, State University can store 200 files on hard disk, 100 files in computer memory, and 300 files on tape. Users want to store 300 word-processing files, 100 packaged-program files, and 100 data files. Each month a typical word-processing file is accessed eight times; a typical packaged-program file, four times; and a typical data file, two times. When a file is accessed, the time it takes for the file to be retrieved depends on the type of file and on the storage medium (see Table 68).

a If the goal is to minimize the total time per month that users spend accessing their files, formulate a balanced transportation problem that can be used to determine where files should be stored.

b Use the minimum cost method to find a bfs.

c Use the transportation simplex to find an optimal solution.

6 The Gotham City police have just received three calls for police. Five cars are available. The distance (in city blocks) of each car from each call is given in Table 69. Gotham City wants to minimize the total distance cars must travel to respond to the three police calls. Use the Hungarian method to determine which car should respond to which call.

7 There are three school districts in the town of Busville. The number of black and white students in each district are shown in Table 70. The Supreme Court requires the schools in Busville to be racially balanced. Thus, each school must have exactly 300 students, and each school must have the same number of black students. The distances between districts are shown in Table 70.

TABLE 68

Storage Medium	Time (Minutes) Word Processing	Packaged Program	Data
Hard disk	5	4	4
Memory	2	1	1
Tape	10	8	6

TABLE 69

Car	Distance (Blocks) Call 1	Call 2	Call 3
1	10	11	18
2	6	7	7
3	7	8	5
4	5	6	4
5	9	4	7

[†]This problem is based on Evans (1984).

TABLE 70

	No. of Students		Distance to (Miles)	
District	Whites	Blacks	District 2	District 3
1	210	120	3	5
2	210	30	—	4
3	180	150	—	—

Formulate a balanced transportation problem that can be used to determine the minimum total distance that students must be bused while still satisfying the Supreme Court's requirements. Assume that a student who remains in his or her own district will not be bused.

8 Using the northwest corner method to find a bfs, find (via the transportation simplex) an optimal solution to the transportation (minimization) problem shown in Table 71.

9 Solve the following LP:

$$
\begin{aligned}
\min z &= 2x_1 + 3x_2 + 4x_3 + 3x_4 \\
\text{s.t.} \quad & x_1 + x_2 \le 4 \\
& x_3 + x_4 \le 5 \\
& x_1 + x_3 \ge 3 \\
& x_2 + x_4 \ge 6 \\
\min\ & x_j \ge 0 \quad (j = 1, 2, 3, 4)
\end{aligned}
$$

10 Find the optimal solution to the balanced transportation problem in Table 72 (minimization).

11 In Problem 10, suppose we increase s_i to 16 and d_3 to 11. The problem is still balanced, and because 31 units (instead of 30 units) must be shipped, one would think that the total shipping costs would be increased. Show that the total shipping cost has actually decreased by \$2, however. This is called the "more for less" paradox. Explain why increasing both the supply and the demand has decreased cost. Using the theory of shadow prices, explain how one could have predicted that increasing s_1 and d_3 by 1 would decrease total cost by \$2.

12 Use the northwest corner method, the minimum-cost method, and Vogel's method to find basic feasible solutions to the transportation problem in Table 73.

13 Find the optimal solution to Problem 12.

TABLE 71

	12	14	16	60
	14	13	19	50
	17	15	18	40
	40	70	10	

TABLE 72

4	2	4	15
12	8	4	15
10	10	10	

TABLE 73

20	11	3	6	5
5	9	10	2	10
18	7	4	1	15
3	3	12	12	

14 Oilco has oil fields in San Diego and Los Angeles. The San Diego field can produce 500,000 barrels per day, and the Los Angeles field can produce 400,000 barrels per day. Oil is sent from the fields to a refinery, either in Dallas or in Houston (assume that each refinery has unlimited capacity). It costs \$700 to refine 100,000 barrels of oil at Dallas and \$900 at Houston. Refined oil is shipped to customers in Chicago and New York. Chicago customers require 400,000 barrels per day of refined oil; New York customers require 300,000. The costs of shipping 100,000 barrels of oil (refined or unrefined) between cities are given in Table 74. Formulate a balanced transportation model of this situation.

15 For the Powerco problem, find the range of values of c_{24} for which the current basis remains optimal.

16 For the Powerco problem, find the range of values of c_{23} for which the current basis remains optimal.

17 A company produces cars in Atlanta, Boston, Chicago, and Los Angeles. The cars are then shipped to warehouses in Memphis, Milwaukee, New York City, Denver, and San Francisco. The number of cars available at each plant is given in Table 75.

Each warehouse needs to have available the number of cars given in Table 76.

The distance (in miles) between the cities is given in Table 77.

a Assuming that the cost (in dollars) of shipping a car equals the distance between two cities, determine an optimal shipping schedule.

b Assuming that the cost (in dollars) of shipping a car equals the square root of the distance between two cities, determine an optimal shipping schedule.

TABLE 74

	To (\$)			
From	Dallas	Houston	N.Y.	Chicago
L.A.	300	110	—	—
San Diego	420	100	—	—
Dallas	—	—	450	550
Houston	—	—	470	530

TABLE 75

Plant	Cars Available
Atlanta	5,000
Boston	6,000
Chicago	4,000
L.A.	3,000

TABLE 76

Warehouse	Cars Required
Memphis	6,000
Milwaukee	4,000
N.Y.	4,000
Denver	2,000
San Francisco	2,000

TABLE 77

	Memphis	Milwaukee	N.Y.	Denver	S.F.
Atlanta	371	761	841	1,398	2,496
Boston	1,296	1,050	206	1,949	3,095
Chicago	530	87	802	996	2,142
L.A.	1,817	2,012	2,786	1,059	379

18 During the next three quarters, Airco faces the following demands for air conditioner compressors: quarter 1—200; quarter 2—300; quarter 3—100. As many as 240 air compressors can be produced during each quarter. Production costs/compressor during each quarter are given in Table 78. The cost of holding an air compressor in inventory is \$100/quarter. Demand may be backlogged (as long as it is met by the end of quarter 3) at a cost of \$60/compressor/quarter. Formulate the tableau for a balanced transportation problem whose solution tells Airco how to minimize the total cost of meeting the demands for quarters 1–3.

19 A company is considering hiring people for four types of jobs. It would like to hire the number of people in Table 79 for each type of job.

Four types of people can be hired by the company. Each type is qualified to perform two types of jobs according to

TABLE 78

Quarter 1	Quarter 2	Quarter 3
$200	$180	$240

TABLE 79

	Job			
	1	2	3	4
Number of people	30	30	40	20

TABLE 80

	Type of Person			
	1	2	3	4
Jobs qualified for	1 and 3	2 and 3	3 and 4	1 and 4

Table 80. A total of 20 Type 1, 30 Type 2, 40 Type 3, and 20 Type 4 people have applied for jobs. Formulate a balanced transportation problem whose solution will tell the company how to maximize the number of employees assigned to suitable jobs. (*Note:* Each person can be assigned to at most one job.)

20 During each of the next two months you can produce as many as 50 units/month of a product at a cost of $12/unit during month 1 and $15/unit during month 2. The customer is willing to buy as many as 60 units/month during each of the next two months. The customer will pay $20/unit during month 1, and $16/unit during month 2. It costs $1/unit to hold a unit in inventory for a month. Formulate a balanced transportation problem whose solution will tell you how to maximize profit.

Group B

21† The Carter Caterer Company must have the following number of clean napkins available at the beginning of each of the next four days: day 1—15; day 2—12; day 3—18; day 4—6. After being used, a napkin can be cleaned by one of two methods: fast service or slow service. Fast service costs 10¢ per napkin, and a napkin cleaned via fast service is available for use the day after it is last used. Slow service costs 6¢ per napkin, and these napkins can be reused two days after they are last used. New napkins can be purchased for a cost of 20¢ per napkin. Formulate a balanced transportation problem to minimize the cost of meeting the demand for napkins during the next four days.

22 Braneast Airlines must staff the daily flights between New York and Chicago shown in Table 81. Each of Braneast's crews lives in either New York or Chicago. Each day a crew must fly one New York–Chicago and one Chicago–New York flight with at least 1 hour of downtime between flights. Braneast wants to schedule the crews to minimize the total downtime. Set up an assignment problem that can be used to accomplish this goal. (*Hint:* Let $x_{ij} = 1$ if the crew that flies flight i also flies flight j, and $x_{ij} = 0$ otherwise. If $x_{ij} = 1$, then a cost c_{ij} is incurred, corresponding to the downtime associated with a crew flying flight i and flight j.) Of course, some assignments are not possible. Find the flight assignments that minimize the total downtime. How many crews should be based in each city? Assume that at the end of the day, each crew must be in its home city.

†This problem is based on Jacobs (1954).

TABLE 81

Flight	Leave Chicago	Arrive New York	Flight	Leave New York	Arrive Chicago
1	6 A.M.	10 A.M.	1	7 A.M.	9 A.M.
2	9 A.M.	1 P.M.	2	8 A.M.	10 A.M.
3	12 noon	4 P.M.	3	10 A.M.	12 noon
4	3 P.M.	7 P.M.	4	12 noon	2 P.M.
5	5 P.M.	9 P.M.	5	2 P.M.	4 P.M.
6	7 P.M.	11 P.M.	6	4 P.M.	6 P.M.
7	8 P.M.	12 midnight	7	6 P.M.	8 P.M.

23 A firm producing a single product has three plants and four customers. The three plants will produce 3,000, 5,000, and 5,000 units, respectively, during the next time period. The firm has made a commitment to sell 4,000 units to customer 1, 3,000 units to customer 2, and at least 3,000 units to customer 3. Both customers 3 and 4 also want to buy as many of the remaining units as possible. The profit associated with shipping a unit from plant i to customer j is given in Table 82. Formulate a balanced transportation problem that can be used to maximize the company's profit.

24 A company can produce as many as 35 units/month. The demands of its primary customers must be met on time each month; if it wishes, the company may also sell units to secondary customers each month. A $1/unit holding cost is assessed against each month's ending inventory. The relevant data are shown in Table 83. Formulate a balanced transportation problem that can be used to maximize profits earned during the next three months.

25 My home has four valuable paintings that are up for sale. Four customers are bidding for the paintings. Customer 1 is willing to buy two paintings, but each other customer is willing to purchase at most one painting. The prices that each customer is willing to pay are given in Table 84. Use

TABLE 82

	To Customer ($)			
From	1	2	3	4
Plant 1	65	63	62	64
Plant 2	68	67	65	62
Plant 3	63	60	59	60

TABLE 83

Month	Production Cost/Unit ($)	Primary Demand	Available for Secondary Demand	Sales Price/Unit ($)
1	13	20	15	15
2	12	15	20	14
3	13	25	15	16

TABLE 84

Customer	Bid for ($)			
	Painting 1	Painting 2	Painting 3	Painting 4
1	8	11	—	—
2	9	13	12	7
3	9	—	11	—
4	—	—	12	9

the Hungarian method to determine how to maximize the total revenue received from the sale of the paintings.

26 Powerhouse produces capacitors at three locations: Los Angeles, Chicago, and New York. Capacitors are shipped from these locations to public utilities in five regions of the country: northeast (NE), northwest (NW), midwest (MW), southeast (SE), and southwest (SW). The cost of producing and shipping a capacitor from each plant to each region of the country is given in Table 85. Each plant has an annual production capacity of 100,000 capacitors. Each year, each region of the country must receive the following number of capacitors: NE, 55,000; NW, 50,000; MW, 60,000; SE, 60,000; SW, 45,000. Powerhouse feels shipping costs are too high, and the company is therefore considering building one or two more production plants. Possible sites are Atlanta and Houston. The costs of producing a capacitor and shipping it to each region of the country are given in Table 86. It costs $3 million (in current dollars) to build a new plant, and operating each plant incurs a fixed cost (in addition to variable shipping and production costs) of $50,000 per year. A plant at Atlanta or Houston will have the capacity to produce 100,000 capacitors per year.

Assume that future demand patterns and production costs will remain unchanged. If costs are discounted at a rate of $11\frac{1}{9}$% per year, how can Powerhouse minimize the present value of all costs associated with meeting current and future demands?

TABLE 85

From	To ($)				
	NE	NW	MW	SE	SW
L.A.	27.86	4.00	20.54	21.52	13.87
Chicago	8.02	20.54	2.00	6.74	10.67
N.Y.	2.00	27.86	8.02	8.41	15.20

TABLE 86

From	To ($)				
	NE	NW	MW	SE	SW
Atlanta	8.41	21.52	6.74	3.00	7.89
Houston	15.20	13.87	10.67	7.89	3.00

27† During the month of July, Pittsburgh resident B. Fly must make four round-trip flights between Pittsburgh and Chicago. The dates of the trips are as shown in Table 87. B. Fly must purchase four round-trip tickets. Without a discounted fare, a round-trip ticket between Pittsburgh and Chicago costs $500. If Fly's stay in a city includes a weekend, then he gets a 20% discount on the round-trip fare. If his stay in a city is at least 21 days, then he receives a 35% discount; and if his stay is more than 10 days, then he receives a 30% discount. Of course, only one discount can be applied toward the purchase of any ticket. Formulate and solve an assignment problem that minimizes the total cost of purchasing the four round-trip tickets. (*Hint:* Let $x_{ij} = 1$ if a round-trip ticket is purchased for use on the ith flight out of Pittsburgh and the jth flight out of Chicago. Also think about where Fly should buy a ticket if, for example, $x_{21} = 1$.)

28 Three professors must be assigned to teach six sections of finance. Each professor must teach two sections of finance, and each has ranked the six time periods during which finance is taught, as shown in Table 88. A ranking of 10 means that the professor wants to teach that time, and a ranking of 1 means that he or she does not want to teach at that time. Determine an assignment of professors to sections that will maximize the total satisfaction of the professors.

29‡ Three fires have just broken out in New York. Fires 1 and 2 each require two fire engines, and fire 3 requires three fire engines. The "cost" of responding to each fire depends on the time at which the fire engines arrive. Let t_{ij} be the time (in minutes) when the jth engine arrives at fire i. Then the cost of responding to each fire is as follows:

$$\text{Fire 1:}\quad 6t_{11} + 4t_{12}$$
$$\text{Fire 2:}\quad 7t_{21} + 3t_{22}$$
$$\text{Fire 3:}\quad 9t_{31} + 8t_{32} + 5t_{33}$$

Three fire companies can respond to the three fires. Company 1 has three engines available, and companies 2

TABLE 87

Leave Pittsburgh	Leave Chicago
Monday, July 1	Friday, July 5
Tuesday, July 9	Thursday, July 11
Monday, July 15	Friday, July 19
Wednesday, July 24	Thursday, July 25

†Based on Hansen and Wendell (1982).

‡Based on Denardo, Rothblum, and Swersey (1988).

TABLE 88

Professor	9 A.M.	10 A.M.	11 A.M.	1 P.M.	2 P.M.	3 P.M.
1	8	7	6	5	7	6
2	9	9	8	8	4	4
3	7	6	9	6	9	9

TABLE 89

Company	Fire 1	Fire 2	Fire 3
1	6	7	9
2	5	8	11
3	6	9	10

and 3 each have two engines available. The time (in minutes) it takes an engine to travel from each company to each fire is shown in Table 89.

a Formulate and solve a transportation problem that can be used to minimize the cost associated with assigning the fire engines. (*Hint:* Seven demand points will be needed.)

b Would the formulation in part (a) still be valid if the cost of fire 1 were $4t_{11} + 6t_{12}$?

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