Set up Inventory Problem as Transportation Problem

Consider the "SailCo" problem, where we want to determine the number of boats to build in order to satisfy our customer demand. We're given the following information:

- The demand for the next four quarters: 40, 60, 75, 25 (so total demand is 200 boats).
- At the beginning of the first quarter, we have 10.
- During a quarter, we can produce up to 40 sailboats using regular-time labor at a cost of $400 each.
- During a quarter, we can produce additional boats using overtime-labor at a cost of $450 per boat.
- At the end of each quarter, there is a carrying cost (or holding cost) of $20 per boat.

To set this up as a transportation problem, we need to come up with **supply** points and **demand** points:

- The output of each quarter is a "supply", but we do need to distinguish between regular labor and overtime labor. Therefore, there are 4 supply variables for regular labor (one each quarter), and 4 supply points for overtime labor, for a total of 8 supply points.

  But wait! We have to take into account the inventory at the beginning of the first quarter, which can be thought of as one extra supply point (so 9 supply points total).

  Summary: We can use variables $s_0, s_{11}, s_{12}, \ldots, s_{41}, s_{42}$.

- The "demand" for each quarter is straightforward- one demand "point" for each quarter.

  Summary: d_1, d_2, d_3, d_4.

Next, we need to make the problem balanced:

- The total demand is 200 units.
- **We can't currently compute the total supply**, because we can make any number of boats during overtime. We could just pick a very large number, but consider this: If $s_1 = 10$ and $s_2 = 40$, then making 150 boats in overtime during quarter 1 would finish the problem (all over s values are 0), so therefore, we'll make each overtime value equal to 150.

Now let's write down all our variables, where $s_0 = 10$.

$s_{11} = \text{1st qtr reg} = 40$	$s_{31} = \text{3d qtr reg} = 40$	$d_1 = \text{1st qtr demand} = 40$
$s_{12} = \text{1st qtr OT} = 150$	$s_{32} = \text{3d qtr OT} = 150$	$d_2 = \text{2d qtr demand} = 60$
$s_{21} = \text{2d qtr reg} = 40$	$s_{41} = \text{4th qtr reg} = 40$	$d_3 = \text{3d qtr demand} = 75$
$s_{22} = \text{2d qtr OT} = 150$	$s_{42} = \text{4th qtr OT} = 150$	$d_4 = \text{4th qtr demand} = 25$

Finally, to **balance the problem**, we see that we currently have a total supply of 770 units, and a current total demand of 200 units. We'll need a dummy demand of $770 - 200 = 570$ units.

Now consider the first "supply" row. The costs are the carrying costs only:

	d_1	d_2	d_3	d_4	Dummy	Supply
s_0	0	20	40	60	0	10

Finishing up the quarter; cost is 400 per boat plus carrying costs, or 450 per boat plus carrying costs.

	d_1	d_2	d_3	d_4	Dummy	Supply
s_0	0	20	40	60	0	10
s_{11}	400	420	440	460	0	40
s_{12}	450	470	490	510	0	150

Now for the second quarter, we do something similar:

	d_1	d_2	d_3	d_4	Dummy	Supply
s_{21}	?	400	420	440	0	40
s_{22}	?	450	470	490	0	150

What should be in the first column? There may be cells that we want to be sure are never used. In those cells, we make the cost prohibitive- Use M in place of a very very large number. In fact, here is the full table.

	d_1	d_2	d_3	d_4	Dummy	Supply
s_0	0 10	20	40	60	0	10
s_{11}	400 30	420 10	440	460	0	40
s_{12}	450	470	490	510	0 150	150
s_{21}	M	400 40	420	440	0	40
s_{22}	M	450 10	470	490	0 140	150
s_{31}	M	M	400 40	420	0	40
s_{32}	M	M	450 35	470	0 115	150
s_{41}	M	M	M	400 25	0 15	40
s_{42}	M	M	M	450	0 150	150
	40	60	75	25	570	770

Shown is the optimal solution found in LibreOffice Calc (gives a minimum value of \$78450.00.