Consider a minimization problem and how we solve it currently:

$$\begin{aligned} \min \quad & w = y_1 + 2y_2 \\ \text{st} \quad & y_1 - 2y_2 + y_3 \geq 4 \\ & 2y_1 + y_2 - y_3 \geq 6 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

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y_1	y_2	y_3	e_1	e_2	*rhs*
1	2	0	0	0	0
1	−2	1	−1	0	4
2	1	−1	0	−1	6

How to proceed?

Consider a minimization problem and how we solve it currently:

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y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
1	−2	1	−1	0	4
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How to proceed?

Big M or Two Phase

What if we do this:

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
1	-2	1	-1	0	4
2	1	-1	0	-1	6

What if we do this:

$$
\begin{array}{ccccc|c}
y_1 & y_2 & y_3 & e_1 & e_2 & rhs \\
1 & 2 & 0 & 0 & 0 & 0 \\
\hline
1 & -2 & 1 & -1 & 0 & 4 \\
2 & 1 & -1 & 0 & -1 & 6
\end{array}
\rightarrow
\begin{array}{ccccc|c}
y_1 & y_2 & y_3 & e_1 & e_2 & rhs \\
1 & 2 & 0 & 0 & 0 & 0 \\
\hline
-1 & 2 & -1 & 1 & 0 & -4 \\
-2 & -1 & 1 & 0 & 1 & -6
\end{array}
$$

What if we do this:

$$\begin{array}{rrrrr|r} y_1 & y_2 & y_3 & e_1 & e_2 & rhs \\ 1 & 2 & 0 & 0 & 0 & 0 \\ \hline 1 & -2 & 1 & -1 & 0 & 4 \\ 2 & 1 & -1 & 0 & -1 & 6 \end{array} \rightarrow \begin{array}{rrrrr|r} y_1 & y_2 & y_3 & e_1 & e_2 & rhs \\ 1 & 2 & 0 & 0 & 0 & 0 \\ \hline -1 & 2 & -1 & 1 & 0 & -4 \\ -2 & -1 & 1 & 0 & 1 & -6 \end{array}$$

Is the current solution *basic*?

What if we do this:

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
1	-2	1	-1	0	4
2	1	-1	0	-1	6

$\rightarrow$

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
-1	2	-1	1	0	-4
-2	-1	1	0	1	-6

Is the current solution *basic*? Yes.

What if we do this:

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
1	-2	1	-1	0	4
2	1	-1	0	-1	6

$\rightarrow$

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
-1	2	-1	1	0	-4
-2	-1	1	0	1	-6

Is the current solution *basic*? Yes.

Is the current solution *feasible*?

What if we do this:

y_1	y_2	y_3	e_1	e_2	*rhs*
1	2	0	0	0	0
1	−2	1	−1	0	4
2	1	−1	0	−1	6

$\rightarrow$

y_1	y_2	y_3	e_1	e_2	*rhs*
1	2	0	0	0	0
−1	2	−1	1	0	−4
−2	−1	1	0	1	−6

Is the current solution *basic*? Yes.

Is the current solution *feasible*? No.

What if we do this:

$$\begin{array}{ccccc|c} y_1 & y_2 & y_3 & e_1 & e_2 & rhs \\ 1 & 2 & 0 & 0 & 0 & 0 \\ \hline 1 & -2 & 1 & -1 & 0 & 4 \\ 2 & 1 & -1 & 0 & -1 & 6 \end{array} \rightarrow \begin{array}{ccccc|c} y_1 & y_2 & y_3 & e_1 & e_2 & rhs \\ 1 & 2 & 0 & 0 & 0 & 0 \\ \hline -1 & 2 & -1 & 1 & 0 & -4 \\ -2 & -1 & 1 & 0 & 1 & -6 \end{array}$$

Is the current solution *basic*? Yes.

Is the current solution *feasible*? No.

We cannot start our usual simplex algorithm.

However, consider the dual:

$$\begin{array}{rl} \min & w = y_1 + 2y_2 \\ \text{st} & y_1 - 2y_2 + y_3 \geq 4 \\ & 2y_1 + y_2 - y_3 \geq 6 \\ & y_1, y_2, y_3 \geq 0 \end{array} \quad \Rightarrow$$

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And the tableaux:

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
−1	2	−1	1	0	−4
−2	−1	1	0	1	−6

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$$\begin{array}{rl} \min & w = y_1 + 2y_2 \\ \text{st} & y_1 - 2y_2 + y_3 \geq 4 \\ & 2y_1 + y_2 - y_3 \geq 6 \\ & y_1, y_2, y_3 \geq 0 \end{array} \quad \Rightarrow \quad \begin{array}{rrl} \max & z = 4x_1 + 6x_2 & \\ \text{st} & x_1 + 2x_2 & \leq 1 \\ & -2x_1 + x_2 & \leq 2 \\ & x_1 - x_2 & \leq 0 \\ & x_1, x_2 \geq 0 & \end{array}$$

And the tableaux:

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
-1	2	-1	1	0	-4
-2	-1	1	0	1	-6

x_1	x_2	s_1	s_2	s_3	rhs
-4	-6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
-1	-1	0	0	1	0

However, consider the dual:

$$\begin{array}{ll} \min & w = y_1 + 2y_2 \\ \text{st} & y_1 - 2y_2 + y_3 \geq 4 \\ & 2y_1 + y_2 - y_3 \geq 6 \\ & y_1, y_2, y_3 \geq 0 \end{array} \quad \Rightarrow \quad \begin{array}{lrl} \max & z = 4x_1 + 6x_2 & \\ \text{st} & x_1 + 2x_2 & \leq 1 \\ & -2x_1 + x_2 & \leq 2 \\ & x_1 - x_2 & \leq 0 \\ & x_1, x_2 \geq 0 & \end{array}$$

And the tableaux:

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
−1	2	−1	1	0	−4
−2	−1	1	0	1	−6

x_1	x_2	s_1	s_2	s_3	rhs
−4	−6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
−1	−1	0	0	1	0

The tableau on the right: We have a BFS.
The tableau on the left: Basic, but not feasible.

First step in the simplex method: Col w/largest neg, Row 0

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x_1	x_2	s_1	s_2	s_3	rhs
-4	-6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
-1	-1	0	0	1	0

Second step, Simplex: “ratio test” for pivot row.

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x_1	x_2	s_1	s_2	s_3	rhs
-4	-6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
-1	-1	0	0	1	0

Second step, Simplex: "ratio test" for pivot row.

x_1	x_2	s_1	s_2	s_3	rhs
-4	-6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
-1	-1	0	0	1	0

What do these two steps mean for the dual?

x_1	x_2	s_1	s_2	s_3	rhs	
-4	-6	0	0	0	0	
1	2	1	0	0	1	$\rightarrow$
2	1	0	1	0	2	
-1	-1	0	0	1	0	

x_1	x_2	s_1	s_2	s_3	rhs
-4	-6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
-1	-1	0	0	1	0

$\rightarrow$

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
-1	2	-1	1	0	-4
-2	-1	1	0	1	-6

Largest negative, Row 0 (Primal) $\rightarrow$ Largest negative, RHS (Dual)

x_1	x_2	s_1	s_2	s_3	rhs
-4	-6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
-1	-1	0	0	1	0

$\rightarrow$

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
-1	2	-1	1	0	-4
-2	-1	1	0	1	-6

Largest negative, Row 0 (Primal) $\rightarrow$ Largest negative, RHS (Dual)

Ratio Test, Primal: b_i/a_{ik}
$1/2, 2/1, (0/-1)$

$\rightarrow$

Ratio Test, Dual: c_i/a_{ki}
$|1/(-2)|, |2/(-1)|, (0/1)$

x_1	x_2	s_1	s_2	s_3	rhs
-4	-6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
-1	-1	0	0	1	0

$\rightarrow$

x_1	x_2	s_1	s_2	s_3	rhs
−4	−6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
−1	−1	0	0	1	0

$\rightarrow$

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
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−2	−1	1	0	1	−6

Largest negative, Row 0 (Primal) $\rightarrow$ Largest negative, RHS (Dual)

Ratio Test, Primal: b_i/a_{ik} $\rightarrow$ Ratio Test, Dual: c_i/a_{ki}

$1/2, 2/1, (0/-1)$ $\rightarrow$ $|1/(-2)|, |2/(-1)|, (0/1)$

x_1	x_2	s_1	s_2	s_3	rhs
−4	−6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
−1	−1	0	0	1	0

$\rightarrow$

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
−1	2	−1	1	0	−4
−2	−1	1	0	1	−6

Row Reduce both Primal and Dual

(Primal) Ratio of RHS to pos col value, take the min.
(Dual) Ratio of Row 0 to neg row value, take min (abs value)

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x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	*rhs*
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

The current basic solution is:

Row Reduce both Primal and Dual

(Primal) Ratio of RHS to pos col value, take the min.
(Dual) Ratio of Row 0 to neg row value, take min (abs value)

x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	*rhs*
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

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x_1	x_2	s_1	s_2	s_3
0	$1/2$	0	$3/2$	$1/2$

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-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	*rhs*
0	$3/2$	$1/2$	0	$1/2$	-3
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1	$1/2$	$1/2$	0	$-1/2$	3

The current basic solution is:

x_1	x_2	s_1	s_2	s_3	
0	$1/2$	0	$3/2$	$1/2$	Feasible, not optimal
e_1	e_2	y_1	y_2	y_3	
-1	0	3	0	0	Basic, Not feasible

Row Reduce both Primal and Dual

(Primal) Ratio of RHS to pos col value, take the min.
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x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	*rhs*
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

The current basic solution is:

x_1	x_2	s_1	s_2	s_3	
0	$1/2$	0	$3/2$	$1/2$	Feasible, not optimal
e_1	e_2	y_1	y_2	y_3	
-1	0	3	0	0	Basic, Not feasible

Next step?

Compare Simplex on Left to the Dual on Right:

x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

Compare Simplex on Left to the Dual on Right:

x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	*rhs*
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

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x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	*rhs*
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

Ratio test gives:

Compare Simplex on Left to the Dual on Right:

x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	rhs
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

Ratio test gives:

x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

Compare Simplex on Left to the Dual on Right:

x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	rhs
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

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x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	rhs
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

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x_1	x_2	s_1	s_2	s_3	
-1	0	3	0	0	3
$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	rhs
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

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x_1	x_2	s_1	s_2	s_3	
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$1/2$	1	$1/2$	0	0	$1/2$
$-5/2$	0	$-1/2$	1	0	$3/2$
$3/2$	0	$1/2$	0	1	$1/2$

y_1	y_2	y_3	e_1	e_2	rhs
0	$3/2$	$1/2$	0	$1/2$	-3
0	$5/2$	$-3/2$	1	$-1/2$	-1
1	$1/2$	$1/2$	0	$-1/2$	3

Now pivot:

Row Reduce:

x_1	x_2	s_1	s_2	s_3	
0	0	10/3	0	2/3	10/3
0	1	1/3	0	−1/3	1/3
0	0	1/3	1	5/3	7/3
1	0	1/3	0	2/3	1/3

y_1	y_2	y_3	e_1	e_2	*rhs*
0	7/3	0	1/3	1/3	−10/3
0	−5/3	1	−2/3	1/3	2/3
1	−1/3	0	−1/3	−1/3	10/3

Row Reduce:

x_1	x_2	s_1	s_2	s_3	
0	0	10/3	0	2/3	10/3
0	1	1/3	0	−1/3	1/3
0	0	1/3	1	5/3	7/3
1	0	1/3	0	2/3	1/3

y_1	y_2	y_3	e_1	e_2	*rhs*
0	7/3	0	1/3	1/3	−10/3
0	−5/3	1	−2/3	1/3	2/3
1	−1/3	0	−1/3	−1/3	10/3

The current basic solution is:

Row Reduce:

x_1	x_2	s_1	s_2	s_3	
0	0	$10/3$	0	$2/3$	$10/3$
0	1	$1/3$	0	$-1/3$	$1/3$
0	0	$1/3$	1	$5/3$	$7/3$
1	0	$1/3$	0	$2/3$	$1/3$

y_1	y_2	y_3	e_1	e_2	*rhs*
0	$7/3$	0	$1/3$	$1/3$	$-10/3$
0	$-5/3$	1	$-2/3$	$1/3$	$2/3$
1	$-1/3$	0	$-1/3$	$-1/3$	$10/3$

The current basic solution is:

y_1	y_2	y_3	e_1	e_2	
$10/3$	0	$2/3$	0	0	Feasible
s_1	s_2	s_3	x_1	x_2	
0	$7/3$	0	$1/3$	$1/3$	Feasible

Therefore optimal.

Remark 1: Back at the Beginning

Given a min, convert to a max, construct the tableau.

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1	2	0	0	0	0
-1	2	-1	1	0	-4
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x_1	x_2	s_1	s_2	s_3	rhs
-4	-6	0	0	0	0
1	2	1	0	0	1
2	1	0	1	0	2
-1	-1	0	0	1	0

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1	2	1	0	0	1
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-1	-1	0	0	1	0

Row 0 ≥ 0 for min $\Rightarrow$ RHS of max ≥ 0 (feasible).

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Conclusion? Infeasible: $2y_1 + y_2 + y_3 + e_2 = -6$
but $y_1, y_2, y_3, e_2 \geq 0$.

Dual Simplex Method can be used for this kind of problem:

y_1	y_2	y_3	e_1	e_2	rhs
1	2	0	0	0	0
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Convert to max, construct tableau.

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4. For each non-negative entry in Row 0, abs ratio test with negative values in Pivot Row (denominator). (Can't do it? Infeasible. Remark 2)
5. Pivot.
6. Repeat from Step 3.

Example

Solve the min problem:

$$\begin{aligned} \min \quad & 2x_1 + 3x_2 + 4x_3 \\ \text{st} \quad & x_1 + 2x_2 + x_3 \geq 3 \\ & 2x_1 - x_2 + 3x_3 \geq 4 \end{aligned}$$

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SOLUTION: Convert to max, write the tableau, multiply constraints by -1.

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SOLUTION: Convert to max, write the tableau, multiply constraints by -1.

x_1	x_2	x_3	e_1	e_2	rhs
2	3	4	0	0	0
1	2	1	-1	0	3
2	-1	3	0	-1	4

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Determine the first pivot position...

Bring in x_1, perform Row Ops:

x_1	x_2	x_3	e_1	e_2	rhs
0	4	1	0	1	-4
0	$-5/2$	$1/2$	1	$-1/2$	-1
1	$-1/2$	$3/2$	0	$-1/2$	2

Bring in x_1, perform Row Ops:

x_1	x_2	x_3	e_1	e_2	rhs
0	4	1	0	1	-4
0	$-5/2$	$1/2$	1	$-1/2$	-1
1	$-1/2$	$3/2$	0	$-1/2$	2

Determine next pivot...

Bring in x_1, perform Row Ops:

x_1	x_2	x_3	e_1	e_2	rhs
0	4	1	0	1	-4
0	$-5/2$	$1/2$	1	$-1/2$	-1
1	$-1/2$	$3/2$	0	$-1/2$	2

Determine next pivot... Bring in x_2:

Bring in x_1, perform Row Ops:

x_1	x_2	x_3	e_1	e_2	rhs
0	4	1	0	1	-4
0	$-5/2$	$1/2$	1	$-1/2$	-1
1	$-1/2$	$3/2$	0	$-1/2$	2

Determine next pivot... Bring in x_2:

x_1	x_2	x_3	e_1	e_2	rhs
0	0	$9/5$	$8/5$	$1/5$	$-28/5$
0	1	$-1/5$	$-2/5$	$1/5$	$2/5$
1	0	$7/5$	$-1/5$	$-2/5$	$11/5$

Bring in x_1, perform Row Ops:

x_1	x_2	x_3	e_1	e_2	rhs
0	4	1	0	1	-4
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Optimal Solution:

Bring in x_1, perform Row Ops:

x_1	x_2	x_3	e_1	e_2	rhs
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1	0	$7/5$	$-1/5$	$-2/5$	$11/5$

Optimal Solution:
$x_1 = 11/5, x_2 = 2/5,\ x_3 = 0$ and $z = 28/5$.

What happens if we want to bring in a new constraint, like $x_1 + 2x_2 \geq 4$?

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Conclusion?

What happens if we want to bring in a new constraint, like $x_1 + 2x_2 \geq 4$?

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0	0	$9/5$	$8/5$	$1/5$	0	$-28/5$
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1	0	$7/5$	$-1/5$	$-2/5$	0	$11/5$
0	0	1	-1	0	1	-1

Conclusion?

Current solution not optimal. (Check constraint as well)

Note that you could indeed bring the last row in as a pivot row and pivot to get a new optimal solution:

x_1	x_2	x_3	e_1	e_2	e_3	rhs
0	0	17/5	0	1/5	8/5	−36/5
0	1	−3/5	0	1/5	−2/5	4/5
1	0	6/5	0	−2/5	−1/5	12/5
0	0	−1	1	0	−1	1

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By adding a new constraint, there are three possible outcomes.

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By adding a new constraint, there are three possible outcomes.

- Current solution satisfies new constraint.
  Conclusion: Old solution still optimal.
- Current solution does not satisfy new constraint, but LP is still feasible.
  Use dual simplex to incorporate the new constraint into tableau and get new solution.
- Current solution does not satisfy new constraint, new LP becomes infeasible.

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- If the RHS of a constraint is changed until it is negative, the solution is no longer feasible.

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Uses of Dual Simplex: New RHS, New Solution

- If the RHS of a constraint is changed until it is negative, the solution is no longer feasible.
- To find the new solution, we had to start from the beginning again.
- Now, use Dual Simplex to incorporate the new constraint into the problem.

Example: Change the RHS, get new solution.

Starting (max) tableau and final tableau:

x_1	x_2	s_1	s_2	rhs
-3	-2	0	0	0
1	1	1	0	4
2	1	0	1	6

Example: Change the RHS, get new solution.

Starting (max) tableau and final tableau:

x_1	x_2	s_1	s_2	rhs
-3	-2	0	0	0
1	1	1	0	4
2	1	0	1	6

$\rightarrow$

x_1	x_2	s_1	s_2	rhs
0	0	1	1	10
0	1	2	-1	2
1	0	-1	1	2

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Starting (max) tableau and final tableau:

x_1	x_2	s_1	s_2	rhs
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By how much can $b_1 = 4$ change?

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Starting (max) tableau and final tableau:

$$\begin{array}{rrrr|r} x_1 & x_2 & s_1 & s_2 & rhs \\ -3 & -2 & 0 & 0 & 0 \\ \hline 1 & 1 & 1 & 0 & 4 \\ 2 & 1 & 0 & 1 & 6 \end{array} \quad \rightarrow \quad \begin{array}{rrrr|r} x_1 & x_2 & s_1 & s_2 & rhs \\ 0 & 0 & 1 & 1 & 10 \\ \hline 0 & 1 & 2 & -1 & 2 \\ 1 & 0 & -1 & 1 & 2 \end{array}$$

By how much can $b_1 = 4$ change?

$$B^{-1}\mathrm{b} + \Delta(B^{-1})_1$$

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0	0	1	1	10
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1	0	-1	1	2

By how much can $b_1 = 4$ change?

$$B^{-1}\mathrm{b} + \Delta (B^{-1})_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \Delta \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

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$$\begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & rhs \\ -3 & -2 & 0 & 0 & 0 \\ \hline 1 & 1 & 1 & 0 & 4 \\ 2 & 1 & 0 & 1 & 6 \end{array} \quad \rightarrow \quad \begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & rhs \\ 0 & 0 & 1 & 1 & 10 \\ \hline 0 & 1 & 2 & -1 & 2 \\ 1 & 0 & -1 & 1 & 2 \end{array}$$

By how much can $b_1 = 4$ change?

$$B^{-1}\mathrm{b} + \Delta(B^{-1})_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \Delta \begin{bmatrix} 2 \\ -1 \end{bmatrix} \geq 0$$

Example: Change the RHS, get new solution.

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x_1	x_2	s_1	s_2	rhs
-3	-2	0	0	0
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$\rightarrow$

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0	0	1	1	10
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By how much can $b_1 = 4$ change?

$$B^{-1}\mathrm{b} + \Delta (B^{-1})_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \Delta \begin{bmatrix} 2 \\ -1 \end{bmatrix} \geq 0$$

We find $-1 \leq \Delta \leq 2$.

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Starting (max) tableau and final tableau:

x_1	x_2	s_1	s_2	rhs
-3	-2	0	0	0
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$\rightarrow$

x_1	x_2	s_1	s_2	rhs
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By how much can $b_1 = 4$ change?

$$B^{-1}\mathrm{b} + \Delta(B^{-1})_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \Delta \begin{bmatrix} 2 \\ -1 \end{bmatrix} \geq 0$$

We find $-1 \leq \Delta \leq 2$. What happens if $\Delta = -2$?

New tableau becomes:

x_1	x_2	s_1	s_2	rhs
0	0	1	1	10
0	1	2	-1	2
1	0	-1	1	2

$\rightarrow$

x_1	x_2	s_1	s_2	rhs
0	0	1	1	8
0	1	2	-1	-2
1	0	-1	1	4

New tableau becomes:

x_1	x_2	s_1	s_2	rhs
0	0	1	1	10
0	1	2	-1	2
1	0	-1	1	2

$\rightarrow$

x_1	x_2	s_1	s_2	rhs
0	0	1	1	8
0	1	2	-1	-2
1	0	-1	1	4

Find the new solution by using the Dual Simplex:

New tableau becomes:

x_1	x_2	s_1	s_2	rhs
0	0	1	1	10
0	1	2	-1	2
1	0	-1	1	2

$\rightarrow$

x_1	x_2	s_1	s_2	rhs
0	0	1	1	8
0	1	2	-1	-2
1	0	-1	1	4

Find the new solution by using the Dual Simplex:

x_1	x_2	s_1	s_2	rhs
0	1	3	0	6
0	-1	-2	1	2
1	1	1	0	2

Example: Mixed Constraints

In this example, we have mixed constraint types and mix regular and dual simplex methods together to solve the tableau.

$$
\begin{array}{rrrl}
\max\ z = & -x_1 & +5x_2 & \\
\text{st} & 2x_1 & -3x_2 & \geq 1 \\
 & x_1 & +x_2 & \leq 3 \\
 & x_1, & x_2 & \geq 0
\end{array}
$$

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x_1	x_2	e_1	s_2	
1	−5	0	0	0
2	−3	−1	0	1
1	1	0	1	3

x_1	x_2	e_1	s_2	
1	−5	0	0	0
−2	3	1	0	−1
1	1	0	1	3

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$$\begin{array}{rrrr|r} x_1 & x_2 & e_1 & s_2 & \\ 1 & -5 & 0 & 0 & 0 \\ \hline -2 & 3 & 1 & 0 & -1 \\ 1 & 1 & 0 & 1 & 3 \end{array}$$

Primal and Dual are both infeasible at this point. Try regular simplex (rule of thumb).

Pivot:

x_1	x_2	e_1	s_2	
1	-5	0	0	0
-2	3	1	0	-1
1	1	0	1	3

Pivot:

$$
\begin{array}{rrrr|r}
x_1 & x_2 & e_1 & s_2 & \\
1 & -5 & 0 & 0 & 0 \\
\hline
-2 & 3 & 1 & 0 & -1 \\
1 & 1 & 0 & 1 & 3
\end{array}
\quad \Rightarrow \quad
\begin{array}{rrrr|r}
x_1 & x_2 & e_1 & s_2 & \\
1 & 0 & 0 & 5 & 15 \\
\hline
-5 & 0 & 1 & -3 & -10 \\
1 & 1 & 0 & 1 & 3
\end{array}
$$

Pivot:

x_1	x_2	e_1	s_2	
1	-5	0	0	0
-2	3	1	0	-1
1	1	0	1	3

$\Rightarrow$

x_1	x_2	e_1	s_2	
1	0	0	5	15
-5	0	1	-3	-10
1	1	0	1	3

Dual feasible. Use dual simplex.
(Find the pivot)

Pivot:

x_1	x_2	e_1	s_2	
1	−5	0	0	0
−2	3	1	0	−1
1	1	0	1	3

$\Rightarrow$

x_1	x_2	e_1	s_2	
1	0	0	5	15
−5	0	1	−3	−10
1	1	0	1	3

Dual feasible. Use dual simplex.
(Find the pivot)

x_1	x_2	e_1	s_2	
0	0	6/5	7/5	3
1	0	−1/5	3/5	2
0	1	1/5	2/5	1

Pivot:

x_1	x_2	e_1	s_2	
1	−5	0	0	0
−2	3	1	0	−1
1	1	0	1	3

$\Rightarrow$

x_1	x_2	e_1	s_2	
1	0	0	5	15
−5	0	1	−3	−10
1	1	0	1	3

Dual feasible. Use dual simplex.
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x_1	x_2	e_1	s_2	
0	0	6/5	7/5	3
1	0	−1/5	3/5	2
0	1	1/5	2/5	1

Feasible for both primal and dual- Optimal. Write the solutions?

Pivot:

x_1	x_2	e_1	s_2	
1	−5	0	0	0
−2	3	1	0	−1
1	1	0	1	3

$\Rightarrow$

x_1	x_2	e_1	s_2	
1	0	0	5	15
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Dual feasible. Use dual simplex.
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x_1	x_2	e_1	s_2	
0	0	6/5	7/5	3
1	0	−1/5	3/5	2
0	1	1/5	2/5	1

Feasible for both primal and dual- Optimal. Write the solutions?

$$x_1 = 2, x_2 = 1$$

Pivot:

x_1	x_2	e_1	s_2	
1	−5	0	0	0
−2	3	1	0	−1
1	1	0	1	3

$\Rightarrow$

x_1	x_2	e_1	s_2	
1	0	0	5	15
−5	0	1	−3	−10
1	1	0	1	3

Dual feasible. Use dual simplex.
(Find the pivot)

x_1	x_2	e_1	s_2	
0	0	6/5	7/5	3
1	0	−1/5	3/5	2
0	1	1/5	2/5	1

Feasible for both primal and dual- Optimal. Write the solutions?

$$x_1 = 2, x_2 = 1 \qquad y_1 = -6/5, y_2 = 7/5$$

Add in a new constraint $x_1 + 3x_2 \leq 3$.

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- Check constraint: $(2) + 3(1) = 5$, so "Case 2".

Add in a new constraint $x_1 + 3x_2 \leq 3$. $\mathcal{B}$ remain?

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- Find the new solution:

x_1	x_2	e_1	s_2	s_3	
0	0	$6/5$	$7/5$	0	3
1	0	$-1/5$	$3/5$	0	2
0	1	$1/5$	$2/5$	0	1
1	3	0	0	1	3

x_1	x_2	e_1	s_2	s_3	
0	0	$6/5$	$7/5$	0	3
1	0	$-1/5$	$3/5$	0	2
0	1	$1/5$	$2/5$	0	1
0	0	$-2/5$	$-9/5$	1	-2

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- Check constraint: $(2) + 3(1) = 5$, so "Case 2".
- Find the new solution:

x_1	x_2	e_1	s_2	s_3	
0	0	$6/5$	$7/5$	0	3
1	0	$-1/5$	$3/5$	0	2
0	1	$1/5$	$2/5$	0	1
1	3	0	0	1	3

x_1	x_2	e_1	s_2	s_3	
0	0	$6/5$	$7/5$	0	3
1	0	$-1/5$	$3/5$	0	2
0	1	$1/5$	$2/5$	0	1
0	0	$-2/5$	$-9/5$	1	-2

Next pivot:

Add in a new constraint $x_1 + 3x_2 \leq 3$. $\mathcal{B}$ remain?

- Check constraint: $(2) + 3(1) = 5$, so "Case 2".
- Find the new solution:

x_1	x_2	e_1	s_2	s_3	
0	0	6/5	7/5	0	3
1	0	−1/5	3/5	0	2
0	1	1/5	2/5	0	1
1	3	0	0	1	3

x_1	x_2	e_1	s_2	s_3	
0	0	6/5	7/5	0	3
1	0	−1/5	3/5	0	2
0	1	1/5	2/5	0	1
0	0	−2/5	−9/5	1	−2

Next pivot: $(3, 4)$

After pivot:

x_1	x_2	e_1	s_2	s_3	
0	0	8/9	0	7/9	13/9
1	0	−1/3	0	1/3	4/3
0	1	1/9	0	2/9	5/9
0	0	2/9	1	−5/9	10/9