Recall the football/soap opera problem. We want to solve the linear program, given the following goals, and given the budget constraint:

- Goal 1: Get at least 40 HIM.
- Goal 2: Get at least 60 LIP.
- Goal 3: Get at least 35 HIW.
This translates to 3 “Row 0’s”

\[
\begin{align*}
\text{min } z &= P_1 s_1 \\
\text{min } z &= P_2 s_2 \\
\text{min } z &= P_3 s_3
\end{align*}
\]

with constraints:

\[
\begin{align*}
7x_1 + 3x_2 + s_1 - e_1 &= 40 \quad \text{HIM} \\
10x_1 + 5x_2 + s_2 - e_2 &= 60 \quad \text{LIP} \\
5x_1 + 4x_2 + s_3 - e_3 &= 35 \quad \text{HIW} \\
100x_1 + 60x_2 &\leq 600 \quad \text{Budget}
\end{align*}
\]
So that the (max) tableau:

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$e_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
<th>rhs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$P_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>$P_2$</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>$P_3$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>−1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>40</td>
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<tr>
<td>10</td>
<td>5</td>
<td>0</td>
<td>−1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>−1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>35</td>
</tr>
<tr>
<td>100</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>600</td>
</tr>
</tbody>
</table>

We need to “clean up” all the objective function rows so that we have columns of the identity.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>rhs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$e_1$</td>
<td>$e_2$</td>
<td>$e_3$</td>
<td>$s_1$</td>
<td>$s_2$</td>
<td>$s_3$</td>
<td>$s_4$</td>
</tr>
<tr>
<td>$-7P_1$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$-10P_2$</td>
<td>$-5P_2$</td>
<td>0</td>
<td>$P_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$-5P_3$</td>
<td>$-4P_3$</td>
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<td>0</td>
<td>$P_3$</td>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>$-1$</td>
<td>0</td>
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<td>1</td>
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<td>0</td>
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</tr>
<tr>
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<td>5</td>
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<td>$-1$</td>
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<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>$-1$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>100</td>
<td>60</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

We focus on the Simplex Method using only the first “Row 0”. First, we pivot in Column 1 (and first constraint row)
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<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$e_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
<th>rhs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$P_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{-5}{7}P_2$</td>
<td>$\frac{-10}{7}P_2$</td>
<td>$P_2$</td>
<td>0</td>
<td>$\frac{10}{7}P_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{-20}{7}P_2$</td>
</tr>
<tr>
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<td>$\frac{-13}{7}P_3$</td>
<td>$\frac{-5}{7}P_3$</td>
<td>0</td>
<td>$P_3$</td>
<td>$\frac{5}{7}P_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{-45}{7}P_3$</td>
</tr>
<tr>
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<td>$\frac{-1}{7}$</td>
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<td>0</td>
<td>$\frac{1}{7}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{40}{7}$</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{5}{7}$</td>
<td>$\frac{10}{7}$</td>
<td>$\frac{-1}{7}$</td>
<td>0</td>
<td>$\frac{-10}{7}$</td>
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<td>$\frac{20}{7}$</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{13}{7}$</td>
<td>$\frac{5}{7}$</td>
<td>0</td>
<td>$\frac{-1}{7}$</td>
<td>$\frac{-5}{7}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{45}{7}$</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{\frac{120}{7}}{7}$</td>
<td>$\frac{\frac{100}{7}}{7}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{-\frac{100}{7}}{7}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$\frac{\frac{200}{7}}{7}$</td>
</tr>
</tbody>
</table>
We focus on the Simplex Method using only the first “Row 0”. First, we pivot in Column 1 (and first constraint row). Pivot in Column 3, and after Ratio Test, use the second constraint (there was a tie between the second and last).
We will now work with our third goal. Pivot in Column 2, last row.
This is the final tableau. We were unable to meet Goal 3 ($S_3 = 5$), but we did meet goals 1 and 2: Use all ad time in football (6 units), and no time in soaps (0).
This is the final tableau. We were unable to meet Goal 3 \((S_3 = 5)\), but we did meet goals 1 and 2: Use all ad time in football (6 units), and no time in soaps (0).
(Exercise 4, 4.16). We have two products, and we have 32 hours of labor available, a goal of 48 profit, and some demand.

\[
\begin{align*}
4x_1 + 2x_2 + s_1 - e_1 &= 32 \text{ Labor} \\
x_1 + s_2 - e_2 &= 7 \text{ Demand 1} \\
x_2 + s_3 - e_3 &= 10 \text{ Demand 2} \\
4x_1 + 2x_2 + s_4 - e_4 &= 48 \text{ Budget goal}
\end{align*}
\]

- Goal 1: Avoid underutilization of labor.
- Goal 2: Meet demand for product 1.
- Goal 3: Meet demand for product 2.
- Goal 4: Do not use any overtime.
From our equations, we get the tableau (max):

\[
\begin{bmatrix}
  x_1 & x_2 & s_1 & s_2 & s_3 & e_1 & e_2 & e_3 & \text{rhs} \\
  0 & 0 & P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & P_2 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & P_3 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & P_4 & 0 & 0 & 0 \\
  4 & 2 & 1 & 0 & 0 & -1 & 0 & 0 & 32 \\
  1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 7 \\
  0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 10 \\
\end{bmatrix}
\]

In this case, we’ll solve in LINGO.
In LINGO, we’ll minimize \( s_1 \) first:

\[
\begin{align*}
\text{min} &= s_1; \\
4x_1 + 2x_2 + s_1 - e_1 &= 32; \\
x_1 + s_2 - e_2 &= 7; \\
x_2 + s_3 - e_3 &= 10; \\
4x_1 + 2x_2 + s_4 - e_4 &= 48;
\end{align*}
\]

LINGO returns \( x_1 = 0 \) and \( x_2 = 16 \). Now we bring in the second constraint:
In LINGO, we’ll minimize $s_1$ first:

\[
\begin{align*}
\text{min} &= s_1; \\
4x_1 + 2x_2 + s_1 - e_1 &= 32; \\
x_1 + s_2 - e_2 &= 7; \\
x_2 + s_3 - e_3 &= 10; \\
4x_1 + 2x_2 + s_4 - e_4 &= 48; \\
\end{align*}
\]

LINGO returns $x_1 = 0$ and $x_2 = 16$. Now we bring in the second constraint:
Set \( s_1 = 0 \) as a new constraint, and minimize \( s_2 \):

\[
\begin{align*}
\text{min} & = s_2; \\
4x_1 + 2x_2 + s_1 - e_1 & = 32; \\
x_1 + s_2 - e_2 & = 7; \\
x_2 + s_3 - e_3 & = 10; \\
4x_1 + 2x_2 + s_4 - e_4 & = 48; \\
s_1 & = 0;
\end{align*}
\]
Set $s_1 = 0$ as a new constraint, and minimize $s_2$:

\[
\begin{align*}
\text{min} &= s_2; \\
4x_1 + 2x_2 + s_1 - e_1 &= 32; \\
x_1 + s_2 - e_2 &= 7; \\
x_2 + s_3 - e_3 &= 10; \\
4x_1 + 2x_2 + s_4 - e_4 &= 48; \\
s_1 &= 0;
\end{align*}
\]

LINGO returns $x_1 = 7$ and $x_2 = 2$. Now bring in the third constraint:
Set $s_2 = 0$ as a new constraint, and minimize $s_3$:

\[
\begin{align*}
\text{min} &= s_3; \\
4x_1 + 2x_2 + s_1 - e_1 &= 32; \\
x_1 + s_2 - e_2 &= 7; \\
x_2 + s_3 - e_3 &= 10; \\
4x_1 + 2x_2 + s_4 - e_4 &= 48; \\
s_1 &= 0; \\
s_2 &= 0;
\end{align*}
\]

LINGO returns $x_1 = 7$ and $x_2 = 10$. At this stage, we won't be able to drive $e_1$ to zero, and this is our optimal solution.
Set $s_2 = 0$ as a new constraint, and minimize $s_3$:

\[
\begin{align*}
\text{min} &= s_3; \\
4x_1 + 2x_2 + s_1 - e_1 &= 32; \\
x_1 + s_2 - e_2 &= 7; \\
x_2 + s_3 - e_3 &= 10; \\
4x_1 + 2x_2 + s_4 - e_4 &= 48; \\
s_1 &= 0; \\
s_2 &= 0;
\end{align*}
\]

LINGO returns $x_1 = 7$ and $x_2 = 10$. At this stage, we won’t be able to drive $e_1$ to zero, and this is our optimal solution.