Review Material, Exam 2, Ops Research

1. Prove the weak duality theorem: For any $\mathbf{x}$ feasible for the primal and $\mathbf{y}$ feasible for the dual, then...

   HINT: Put the primal so that $A\mathbf{x} \leq \mathbf{b}$ and the dual so that $A^T\mathbf{y} \geq \mathbf{c}$

2. Show that the solution to the dual is $\mathbf{y} = (\mathbf{c}_B^T B^{-1})^T$ (if the primal and dual are both feasible).

   HINT: Strong duality might be useful.

3. Solve using big-M:

$$\begin{array}{rl} \max \quad z = & 2x_1 + 3x_2 \\ \text{st} & 2x_1 + x_2 \geq 4 \\ & x_1 - x_2 \geq -1 \\ & x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{array}$$

4. Solve the last problem again using the dual simplex method.

5. Going back to the original LP in (1), if the constraint $x_1 + x_2 <= 4$ is added, does the basis stay optimal? If not, find the new basis (and the new solution).

6. Using the big-M method on a maximization problem, I got the following tableau:

	x_1	x_2	x_3	s_1	e_1	e_2	a_1	a_2	rhs
	$-1/2+2M$	$-5/2+M$	M	$1/2+M$	M	M	0	0	$2-3M$
x_3	$1/2$	$1/2$	1	$1/2$	0	0	0	0	2
a_1	$-3/2$	$-1/2$	0	$-1/2$	-1	0	1	0	2
a_2	$-1/2$	$-1/2$	0	$-1/2$	0	-1	0	1	1

   Should I stop or should I go? If I stop, what should I conclude?

7. Use the two phase method to solve the following

$$\begin{array}{rl} \max \quad z = & x_1 + x_2 \\ \text{st} & x_1 - x_2 - x_3 = 1 \\ & -x_1 + x_2 + 2x_3 - x_4 = 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

8. Consider the LP and the optimal tableau with missing Row 0.

$$\begin{array}{rl} \max z = & 3x_1 + x_2 \\ \text{s.t.} & 2x_1 + x_2 \leq 4 \\ & 3x_1 + 2x_2 \geq 6 \\ & 4x_1 + 2x_2 = 7 \\ & x_1, x_2 \geq 0 \end{array}$$

x_1	x_2	s_1	e_2	a_2	a_3	rhs
0	0	1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
0	1	0	-2	2	$-\frac{3}{2}$	$\frac{3}{2}$
1	0	0	1	-1	1	1

   Find Row 0.

9. Consider the following LP and its optimal tableau:

$$\begin{array}{rl} \max & z = 4x_1 + x_2 + 2x_3 \\ \text{st} & 8x_1 + 3x_2 + x_3 \leq 12 \\ & 6x_1 + x_2 + x_3 \leq 8 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

x_1	x_2	x_3	s_1	s_2	rhs
8	1	0	0	2	16
2	2	0	1	−1	4
6	1	1	0	1	8

(a) Find the range of values for the coefficient of x_3 which keeps the current basis optimal.

(b) Find the range of values for the coefficient of x_1 which keeps the current basis optimal.

(c) Find the range of values for the RHS of each constraint that keeps the current basis optimal.

(d) Write the dual, and solve it using the tableau for the primal (given above).

10. Give an argument why, if the primal is unbounded, then the dual must be infeasible.

Chapter 6 Review Problems:

3, 4 (except 4(b)), 6, 9, 10, 16, 17, 20, 21, 23, 33, 34.