Review Material, Exam 2, Ops Research

The exam will cover sections 4.11-4.16 (except 4.15), and Chapter 6 (except for 6.4, 6.6 and 6.12). The exam will consist of two parts, an in-class portion and a take-home portion.

The details for the take home exam will come later, but you will need to be able to solve some LPs using a combination of Maple and LINDO. The examples on the class website will remain should you want to use any of them as template files.

For review, be sure to go back over the homework, and work through the review set of questions.

Definitions:

Degenerate LP, artificial variable, deviational variable, sensitivity analysis, shadow price, normal form (for an LP), standard form (for an LP).

Theorems:

Be able to use these theorems to justify your answers. The full theorems are not stated below, but there should be enough there as a memory aid- So you might use these to help you remember which is which. You should look these up to see what the full statements are.

- Weak duality: $\mathbf{c}^T\mathbf{x} \leq \mathbf{y}^T A\mathbf{x} \leq \mathbf{b}^T\mathbf{y}$
- Strong duality: Optimal solutions iff $\mathbf{c}^T\mathbf{x} = \mathbf{b}^T\mathbf{y}$.
- Dual Theorem: $\mathbf{y} = (\mathbf{c}_B^T B^{-1})^{-1}$ and $z = w$.
- Theorem: Shadow prices are the solutions to the dual.
- Complementary Slackness Theorem: $s_i y_i = 0$ and $e_j x_j = 0$.

Algorithms

Be able to compute using these algorithms. In particular, understand when to stop and how to interpret what you have back into the LP. You should also know when you would use each of the algorithms. For the take home exam, you might be asked to use one or more of the algorithms on Maple and LINDO.

- Big-M and Two-Phase
- Goal programming via penalties
- Preemptive goal programming
- The Dual Simplex Method

Techniques:

- Be able to perform sensitivity analysis via a graphical analysis, and algebraically.
- Be able to perform sensitivity analysis incorporating the dual.
- Be able to compute the (final) tableau corresponding to a given basis (directly, without going through the Simplex Method).
- Construct the dual.
- Solve the dual using Row 0 of the optimal tableau for the primal.
- Solve the dual using Complementary Slackness.
- Solve the dual using the Dual Simplex Method.
- Use the Dual Simplex Method to check whether or not a new constraint may be added to an LP.

A Summary of Sensitivity Analysis

We look at the kinds of changes we can make. Recall that the assumptions of this kind of analysis are: Assume that only one change at a time is made, and assume we want the current basis to remain optimal.

These are the changes we discussed, in the order that the text has them listed:

1. Change the coefficient corresponding to a **non-basic variable** (NBV).

   Since $z = \mathbf{c}_B^T B^{-1}\mathbf{b}$, then changing a non-basic variable (and keeping the current basis optimal) will not change the optimal solutions- In particular, the solution to the dual, $\mathbf{y} = (\mathbf{c}_B^T B^{-1})^T$.

   Therefore, we only need to check that the new coefficient still makes the dual solution feasible. If the NBV corresponds to the j^{th} column of A, then we only need to check the j^{th} constraint for the dual:

   $$\mathbf{y}^T\mathbf{a}_j \geq c_j$$

   You might notice that this corresponds to checking that the new Row 0 coefficient is non-negative:

   $$\mathbf{c}_B^T B^{-1}\mathbf{a}_j - c_j \geq 0$$

2. Change the coefficient corresponding to a **basic variable** (BV).

   What changes if we change a basic variable instead of a non-basic variable? Suppose we want to change the i^{th} coordinate of $\mathbf{c}_B$, that corresponds to the j^{th} column of A. Then we want to check to see if the Row 0 coefficients corresponding to NBVs are still non-negative. Before using the Row 0 formula, notice that changing coordinate i will be the same as changing $\mathbf{c}_B^T$ to:

   $$\mathbf{c}_B^T + \Delta\vec{e}_i^{\,T}$$

where $\vec{e}_i^T$ is the i^{th} row of the $m \times m$ identity matrix (if A is $m \times n$ with rank m). Substituting this into our Row 0 formula, we get

$$(\mathbf{c}_B^T + \Delta \vec{e}_i^T) B^{-1} A = \mathbf{c}_B^T B^{-1} A + \Delta \vec{e}_i^T B^{-1} A$$

The important quantity here is the one attached to Δ- This is the i^{th} *row of the optimal tableau.*

The following formula is therefore valid *for the coefficients of the non-basic variables of row 0*:

$$\text{Old Row } 0 + \Delta \ \text{ Row } i \text{ of optimal tableau} \geq 0$$

3. Change the **right hand side** of a constraint.

   We've shown this in Exercise 9, 6.3: If we change the i^{th} RHS, then to keep the current basis optimal, we must have:

$$B^{-1}\mathbf{b} + \Delta B^{-1}\vec{e}_i \geq 0 \quad \Rightarrow \quad \text{Old optimal RHS} + \Delta\, i^{\text{th}} \text{ col. of } B^{-1} \geq 0$$

4. Change the **column values** for a non-basic variable.

   Given the solution to the dual, $\mathbf{y}$, if we change a column corresponding to a non-basic variable, then we need to be sure that the corresponding constraint in the dual remains satisfied. In the normal case,

$$\mathbf{y}^T \mathbf{a}_j \geq c_j$$

5. Add a new **"activity"** (or column).

   Same as the last answer; if we have a new coefficient c_j and a new column $\mathbf{a}_j$, then the current basis remains optimal if the corresponding constraint in the dual remains satisfied. Again, in the normal case, this means

$$\mathbf{y}^T \mathbf{a}_j \geq c_j$$

6. Add a new **constraint**: Analysis is done in 6.11, but if the optimal solution satisfies the new constraint, then the optimal solution remains. If not, the current basis is no longer optimal, and we have to find the new optimal basis (using the dual simplex method).