

Final Exam, Operations Research (Fall 13)

This is a take home exam. You may use your text, any class notes and anything on our class website to help, and you may use LINDO, Maple and/or Matlab. You should not look for solutions on the internet, *or get help from other students*. All work should be your own.

If you have questions, you may consult with me, but I may not be able to answer completely.

SOLUTIONS ARE DUE AT MIDNIGHT on Thursday, Dec 19, but I would highly recommend doing them early. Slide your stapled solutions with this cover sheet under my office door.

1. Questions about convexity

- (a) Show that if $y = f(x)$ is a concave function on \mathbb{R}^1 , then $z = 1/f(x)$ is a convex function (assume that $f(x) > 0$).
- (b) Show that if a function's Hessian has both positive and negative entries along the diagonal, then the function is neither convex nor concave.

2. You are the publisher of a new magazine. The variable cost of printing and distributing each weekly copy of the magazine is 25 cents. You are thinking of charging between \$0.50 and \$1.30 per week for the magazine. The estimated numbers of subscribers (in millions) for weekly prices of the magazine are as follows:

Price	0.5	0.8	1.3
Demand (Millions)	2.0	1.2	0.3

Find the price that will maximize the weekly profit from the magazine. You may use Maple to solve this, but be sure to summarize the solution/give a conclusion.

3. Let $f(x, y) = 4x^2 - 4xy + 2y^2$. By hand, apply the Method of Steepest Descent with the initial point $(2, 3)$, using as large a step size as possible at each iteration. Perform two iterations of the algorithm.

You may use a computer to assist in the computations, but I want to see the computational details at each step.

4. For this problem, we'll construct a linear classifier and we'll use Widrow-Hoff. Download the data from the class website, `FinalExamData01.mat`. When you load the data into Matlab, you'll see two arrays. Array X is 400×4 (that is, 400 points in \mathbb{R}^4), and array T is 400×2 . The first 200 points in X belongs to Class 1, the second 200 points in X belongs to class 2.

You may use the `wid_hoff1.m` function on the class website. You should experiment a bit with the learning rate until you find one that gives you a good result.

To visualize the result, suppose that Y is the 400×2 matrix of network outputs. Then the following two plots will give you an idea of how you're doing:

```
subplot(2,1,1)
plot(1:400,T(:,1),1:400,Y(:,1));
subplot(2,1,2)
plot(1:400,T(:,2),1:400,Y(:,2));
```

Write your solution up in a script file, then publish the results and print it off.

5. For this problem, we will use a feedforward neural network to try to classify photos of people based on gender. The data is on the class website, **MenAndWomen2.mat**. Loading this data set will give you a matrix that is 24138×26 . That is, there are 26 photos- Each photo is a column, and was originally 162×149 pixels.

You should use about 14 photos for training and the rest for validation and testing. You will have to construct a vector for your desired output. The first 13 columns correspond to women, and the second 13 columns correspond to men.

The computer will run out of memory if you attempt to train this using Levenberg-Marquardt (**trainlm**), so change the training to gradient descent (**traingdx**).

Finally, think of a way of presenting your results. One way was given in the previous problem, but part of that depends on how you defined your targets.

Write up this part in a script file, and publish the results (then print it out).

Side Remark To see who is in the first photo, you can type:

```
imagesc(reshape(Y(:,1),162,149));
colormap(gray); axis equal; axis off;
```