Example 2, Section 4.16 (Goal Programming)

The Dewright company is considering three new products to replace current models. Primary consideration should be given to three factors:

- Long-run profit of at least 125 million dollars.
- Maintain current employment of 4000 employees.
- Hold the level of capital investment required now for new equipment to less than 55 million dollars.

It will probably not be possible to attain all of these goals simultaneously, so some relative penalty weights were assigned (penalties relative to each other) as shown below. Each new product' contribution to profit, employment and capital is proportional to the rate of production. These contributions (per unit rate of production) are shown in the table below.

Factor	Prod 1	2	3	Goal	(units)	Penalty
Long run profit	12	9	15	≥ 125	(millions dollars)	5
Employment	5	3	4	$= 40$	(hundreds of people)	$2(+), 4(-)$
Capital	5	7	8	≤ 55	(millions dollars)	3

(Let x_1, x_2, x_3 be the production rates of Products 1, 2, 3 respectively). Formulate a system for goal programming.

Example 3: Preemptive Goal Programming, In-Class

Going back to the advertising agency problem, we have several goals:

$$\begin{array}{lll} \min & z = P_1 s_1^- & \\ \min & z = & P_2 s_2^- \\ \min & z = & & P_3 s_3^- \end{array}$$

Now the using constraints:

$$\begin{array}{rll} 7x_1 + 3x_2 + s_1^- - s_1^+ & = 40 & \text{HIM} \\ 10x_1 + 5x_2 + s_2^- - s_2^+ & = 60 & \text{LIP} \\ 5x_1 + 4x_2 + s_3^- - s_3^+ & = 35 & \text{HIW} \\ 100x_1 + 60x_2 & \leq 600 & \text{Budget} \end{array}$$

And our Tableau now has an unusual form (as in the text, a **minimization** problem.

x_1	x_2	s_1^+	s_2^+	s_3^+	s_1^-	s_2^-	s_3^-	s_4	rhs
0	0	0	0	0	P_1	0	0	0	0
0	0	0	0	0	0	P_2	0	0	0
0	0	0	0	0	0	0	P_3	0	0
7	3	-1	0	0	1	0	0	0	40
10	5	0	-1	0	0	1	0	0	60
5	4	0	0	-1	0	0	1	0	35
100	60	0	0	0	0	0	0	1	600

For the solution, see the Maple sheet (it does the same steps as the text).

Final tableau (for the minimization problem)

x_1	x_2	s_1^+	s_2^+	s_3^+	s_1^-	s_2^-	s_3^-	s_4	rhs
0	0	0	0	0	$-P_1$	0	0	0	0
0	$-P_2$	0	$-P_2$	0	0	0	0	$-P_2/10$	0
0	P_3	0	0	$-P_3$	0	0	0	$-P_3/20$	$5P_3$
1	3/5	0	0	0	$-6/7$	0	0	1/100	6
0	-1	0	-1	0	0	1	0	$-1/10$	0
0	1	0	0	-1	0	0	1	$-1/20$	5
0	6/5	1	0	0	-1	0	0	7/100	2

INTERPRETATIONS and CONCLUSIONS:

Example 4: Color TVs and VCRs

This is a modification of Exercise 3 from the text.

> Highland Appliance must determine how many color TVs and VCRs should be stocked. It costs Highland $300 to purchase a TV and $200 to purchase a VCR. A TV requires 3 square yards of storage space, and a VCR requires 1 square yard. A TV will bring $150 in profit, a VCR $100. Highland has set the following goals (in order of importance):
>
> - Goal 1: We have $20,000 to buy TVs and VCR's
> - Goal 2: We need at least $11,000 in profit
> - Goal 3: We have no more than 200 square yards of storage space.
>
> First, formulate a nonpreemptive goal program that makes the penalty for profit 10 times that of the other two goals, and solve with LINDO. Then make them all equal and re-solve (using LINDO).
>
> Secondly, write a preemptive goal program and solve it with Maple.