

## Homework, Section 12.3

Do problems 1, 3, 6, 7, 8, 12 (Exercise 12 should be:  $h = f + g$  instead of  $h = f = g$ ).

Also, the following:

25. Let

$$f(x_1, x_2, x_3) = 3x_1^2 + 6x_1x_3 + x_2^2 - 4x_2x_3 + 8x_3^2$$

Determine the definiteness of the quadratic form for  $f$ . Hint: Write it as  $\frac{1}{2}\mathbf{x}^T H f(0)\mathbf{x}$  (Since the function is second order, the matrix of second partials is constant, so it won't matter where we evaluate them).

26.  $f(x_1, x_2) = 2x_1 - x_2 - x_1^2 + x_1x_2 - x_2^2$

Is  $f$  convex, concave or indefinite (which could mean neither or both).

27. Find the second order Taylor expansion of the function below at the point  $(1, 0)$ :

$$f(x_1, x_2) = x_1^2 - x_2^2 + 3x_1x_2 + 3e^{-x_2}$$

Determine the definiteness of  $Hf(a)$ .

28. Show (using principal minors) that the function has a global minimum, and find it.

$$f(x, y, z) = x^2 + 2y^2 + 3z^2 + 2xy + 2xz + 3$$

29. Graphically solve the following nonlinear program:

$$\begin{array}{ll} \min z = & x^2 + 9y^2 \\ \text{st} & x + 3y \geq 1 \\ & 2x + y \geq 1 \end{array}$$

Hint: Here is some Maple code to draw some levels curves of your objective function, together with the feasible set:

```
with(plots):  
CS:={2*x+y>=1, x+3*y>=1,x>0,y>0};  
A:=inequal(CS,x=0..2,y=0..2):  
B:=contourplot(x^2+9*y^2, x=0..2, y=0..2, contours=[1/5,1,2,4,6]):  
display(A,B);
```