Homework Addendum for 4.1-4.3

1. Show that the intersection of two convex sets is convex.

2. If we define a *hyperplane* as the set of vectors $\mathbf{x}$ so that $\mathbf{c}^T\mathbf{x} - b = 0$ for some constant vector $\mathbf{c}$ and scalar b, then show that a hyperplane is convex.

3. Show that the convex hull of a given set of k vectors is convex.

4. We said that a point $\mathbf{x}$ is a vertex of polyhedron P if there is a vector $\mathbf{c}$ so that $\mathbf{c}^T\mathbf{x} < \mathbf{c}^T\mathbf{y}$ for all $\mathbf{y} \neq \mathbf{x} \in P$. If P is the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$, then show that $(1,0)$ and $(1,1)$ are indeed vertices by finding appropriate vectors $\mathbf{c}$. Hint: Think hyperplane.

5. Find a **convex combination** of the vertices of the polygonal region below for the point $(1,2)$. You may do this all in two dimensions.

$$\begin{array}{rl} x_1+ & x_2 \leq 5 \\ 3x_1+ & 2x_2 \leq 12 \\ x_1, & x_2 \geq 0 \end{array}$$

6. Consider the feasible set:

$$\begin{array}{rl} 2x+ & y \geq 4 \\ x+ & 2y \geq 6 \\ -x & +2y \geq 2 \\ x, & y \geq 0 \end{array}$$

   (a) Write the feasible set in standard form.

   (b) Find a direction of unboundedness, $\mathbf{d}$.

   (c) Write the point corresponding to $(2,4)$ using Theorem 2, page 135. Be sure to pay attention to the appropriate dimensions!