Examples, Big M Method

1. See the Maple file: `BigM_Example01.mw`

   Here is the LP:

$$\begin{array}{rllll} \max z = & -x_1 & +2x_2 & \\ \text{st} & x_1 & +x_2 & \geq 2 \\ & -x_1 & +x_2 & \geq 1 \\ & & x_2 & \leq 3 \end{array}$$

   Initial tableau using Big-M

	x_1	x_2	e_1	e_2	s_1	a_1	a_2	
	1	-2	0	0	0	M	M	0
a_1	1	1	-1	0	0	1	0	2
a_2	-1	1	0	-1	0	0	1	1
s_1	0	1	0	0	1	0	0	3

   Ending tableau:

	x_1	x_2	e_1	e_2	s_1	a_1	a_2	
	1	0	0	0	2	M	M	0
e_2	1	0	0	1	1	0	-1	2
x_2	0	1	0	0	1	0	0	3
e_1	1	0	1	0	1	-1	0	1

   And our optimal solution is given as $x_1 = 0$, $x_2 = 3$, with $z = 6$.

2. See `BigM_Example02.mw` on the class website.

   This is the LP:

$$
\begin{array}{lrrrl}
\max z = & x_1 & +3x_2 & +x_3 & \\
\text{st} & x_1 & +x_2 & +2x_3 & \le 4 \\
 & -x_1 & & +x_3 & \ge 4 \\
 & & & x_3 & \ge 3
\end{array}
$$

   Where the initial tableau is given by:

	x_1	x_2	x_3	s_1	e_1	e_2	a_1	a_2	rhs
	-1	-3	-1	0	0	0	M	M	
s_1	1	1	2	1	0	0	0	0	4
a_1	-1	0	1	0	-1	0	1	0	4
a_2	0	0	1	0	0	-1	0	1	3

   Here's the final tableau, through Maple:

	x_1	x_2	x_3	s_1	e_1	e_2	a_1	a_2	rhs
	$-\frac{1}{2}+2M$	$-\frac{5}{2}+M$	0	$\frac{1}{2}+M$	M	M	0	0	$2-3M$
x_3	$1/2$	$1/2$	1	$1/2$	0	0	0	0	2
a_1	$-3/2$	$-1/2$	0	$-1/2$	-1	0	1	0	2
a_2	$-1/2$	$-1/2$	0	$-1/2$	0	-1	0	1	1

   Since M is an arbitrarily large positive number, all Row 0 Coefficients are now considered non-negative. Additionally, we failed to remove the artificial variables from the BFS, so therefore, **the LP is infeasible**.

3. See `BigM_Example03.mw` from the class website. Here is the LP

$$\begin{array}{llllll} \max z = & x_1 & +x_2 & & & \\ \text{st} & x_1 & -x_2 & -x_3 & & =1 \\ & -x_1 & +x_2 & +2x_3 & -x_4 & =1 \end{array}$$

Construct the tableau, then remove the M's:

x_1	x_2	x_3	x_4	a_1	a_2	
-1	-1	0	0	M	M	0
1	-1	-1	0	1	0	1
-1	1	2	-1	0	1	1

And the final tableau:

	x_1	x_2	x_3	x_4	a_1	a_2	
	0	-2	0	-1	$2+M$	$1+M$	3
x_1	1	-1	0	-1	2	1	3
x_3	0	0	1	-1	1	1	2

The artificial variables can now be safely deleted (they are non-basic variables). We see that normally we would go into the Simplex Method with our current BFS, but we are unable to pivot in either column 2 or column 4. The LP is **unbounded**, which is clear if we write the system of equations: We maximize $z = 3 + x_2 + x_4$ such that:

$$\mathbf{x} = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

so x_2, x_4 can each be increased without bound (and remain feasible).