Review Material, Exam 2, Ops Research

The exam will cover sections 4.12, 4.13 and 4.16 (Big-M, Two-Phase and goal programming) and Chapter 6 (except for 6.4, 6.6 and 6.12). The exam will consist of two parts, an in-class portion and a take-home portion.

The details for the take home exam will come later. The Maple examples on the class website will remain should you want to use any of them as template files.

For review, be sure to go back over the homework, and work through the review set of questions.

Definitions:

Artificial variables, deviational variables, sensitivity analysis, goal programming, shadow price, normal form (for a max and min), standard form (for an LP), the primal, the dual, complementary slackness.

Theorems:

Be able to use these theorems to justify your answers. The full theorems are not stated below, but there should be enough there as a memory aid- So you might use these to help you remember which is which. You should look these up to see what the full statements are.

- Weak duality: $\mathbf{c}^T\mathbf{x} \leq \mathbf{y}^T A\mathbf{x} \leq \mathbf{b}^T\mathbf{y}$
- Strong duality: Optimal solutions iff $\mathbf{c}^T\mathbf{x} = \mathbf{b}^T\mathbf{y}$.
- Dual Theorem: $\mathbf{y} = (\mathbf{c}_B^T B^{-1})^T$ and $z = w$.
- Theorem: Shadow prices are the solutions to the dual.
- Complementary Slackness Theorem: $s_i y_i = 0$ and $e_j x_j = 0$.

Algorithms

Be able to compute using these algorithms. In particular, understand when to stop and how to interpret what you have back into the LP. You should also know when you would use each of the algorithms.

- Big-M and Two-Phase
- Goal programming via penalties
- Preemptive goal programming
- Construction of the Dual- Normal form and not normal.
- The Dual Simplex Method

Techniques:

- Be able to perform sensitivity analysis via a graphical analysis.
- Understand and use the notation we developed. In particular,

$$\begin{array}{c|c} -\mathbf{c}^T & 0 \\ \hline A & \mathbf{b} \end{array} \quad \longrightarrow \quad \begin{array}{c|c} -\mathbf{c}^T + \mathbf{c}_B^T B^{-1} A & \mathbf{c}_B^T B^{-1}\mathbf{b} \\ \hline B^{-1}A & B^{-1}\mathbf{b} \end{array}$$

- Be able to give the optimal tableau given a current basis. That is, be able to use the linear algebra notation to write down how you would compute the optimal tableau.
- Be able to perform sensitivity analysis for a LP using the optimal tableau.
- Be able to perform sensitivity analysis incorporating the dual.
- Be able to compute the (final) tableau corresponding to a given basis (directly, without going through the Simplex Method).
- Solve the dual using:
  - Row 0 of the optimal tableau for the primal.
  - Complementary Slackness.
  - the Dual Simplex Method.
- Use the Dual Simplex Method to check whether or not a new constraint may be added to an LP.

A Summary of Sensitivity Analysis

The assumptions for sensitivity analysis are that we (i) assume that only one change at a time is made, and (ii) assume we want the current basis to remain optimal. What follows is a summary- See your notes if you want to see more detail.

1. Change the coefficient corresponding to a **non-basic variable** (NBV).
   - Direct analysis without the dual: We just subtract Δ from the optimal Row 0 coefficient, $\hat{c}_j$, to check that the new Row 0 coefficient is non-negative: $\hat{c}_j - \Delta > 0$.
   - Using the dual: Check that the new coefficient still satisfies the j^{th} constraint for the dual:
   $$\mathbf{y}^T \mathbf{a}_j \geq (c_j + \Delta)$$
2. Change the coefficient corresponding to a **basic variable** (BV).

   Remember that the BVs are indexed by the order of the columns of the identity, and there is one per row of the constraint.

   The shortcut formula for the NBVs in Row 0:
   $$\text{Optimal Row } 0 + \Delta \ \text{ Row } i \text{ of optimal tableau} > 0$$
   NOTE: The coefficients for the basic variables are simply set to 0.

3. Change the **right hand side** of a constraint: Optim. RHS + Δ i^{th} col. $B^{-1} > 0$

4. Change the **column values** for a non-basic variable. Two ways to compute it:

   - We could "price out" the new column values if we are given B^{-1}. If $\mathbf{a}_j$ is the new j^{th} column, then the corresponding column in the optimal tableau is:

   $$B^{-1}\mathbf{a}_j$$

   And the new Row 0 value in the j^{th} position is $-c_j + \mathbf{c}_B^T B^{-1}\mathbf{a}_j$

   - Given the solution to the dual, $\mathbf{y}$, if we change a column corresponding to a non-basic variable, then we need to be sure that the corresponding constraint in the dual remains satisfied. In the normal case,

   $$\mathbf{y}^T\mathbf{a}_j \geq c_j$$

5. Add a new **"activity"** (or column). Same as the last answer.

6. Add a new **constraint**: We add a new constraint.

   If the current basis is no longer optimal, and we have to find the new optimal basis (typically, using the dual simplex method).