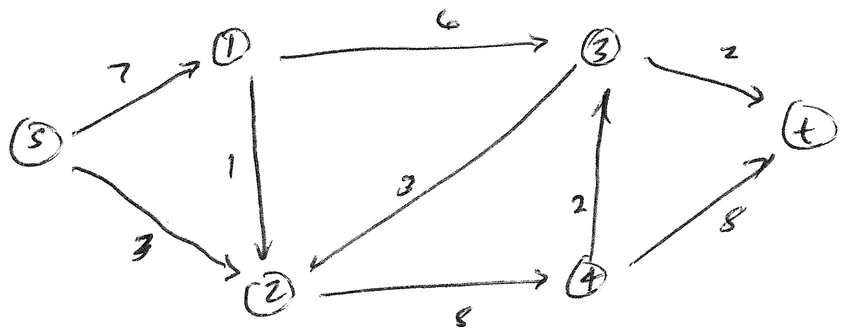
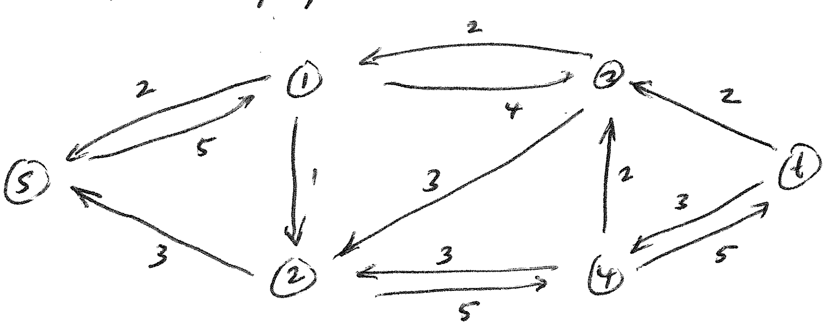
8.3.5

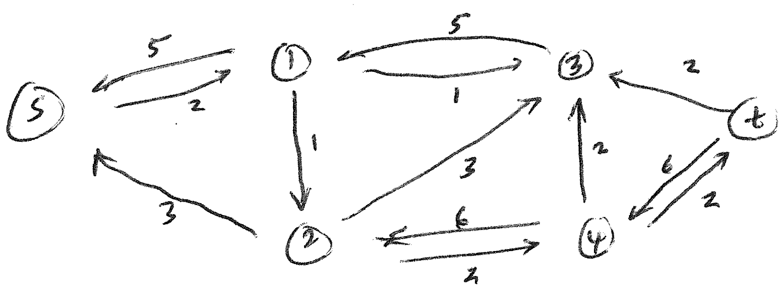



s - 1 - 3 - t has 2 units
s - 2 - 4 - t has 3

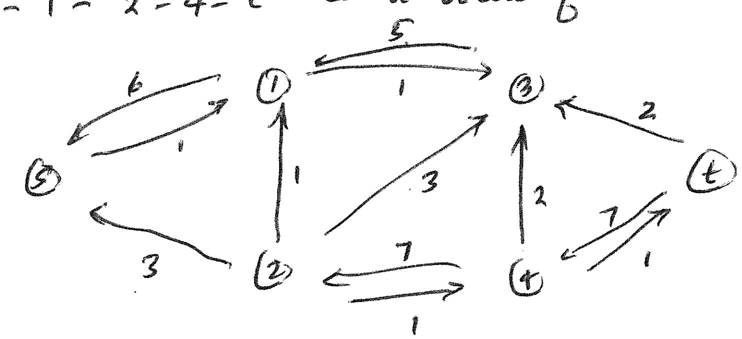
The residual graph:




s - 1 - 3 - 2 - 4 - t has a value of 3




s - 1 - 2 - 4 - t has a value of 1




Done.
Val(f) = 9

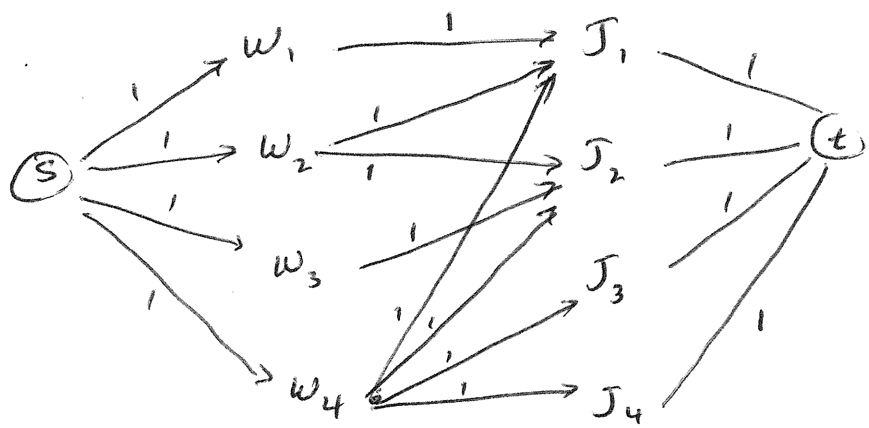
Cut: s - 1 - 3 in A, 2 - 4 - t in B, capacity is 9

8.3.7.

For workers attached to jobs, see figure 8, p. 423

| | Job 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Worker 1 | C | — | — | — |
| 2 | C | C | — | — |
| 3 | — | C | — | — |
| 4 | C | C | C | C |

The network:



12.6.1

The coords for the factories are:

$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ (These are known constants)

Find $(x, y)$ that minimizes:

$$E(x,y) = (x-x_1)^2 + (y-y_1)^2 + (x-x_2)^2 + (y-y_2)^2 + \cdots + (x-x_n)^2 + (y-y_n)^2$$

$$\frac{\partial E}{\partial x} = 2(x-x_1) + 2(x-x_2) + \cdots + 2(x-x_n) = 0$$

Divide by 2, we have: $nx = x_1 + x_2 + \cdots + x_n$

so $x = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

Similarly,

$$\frac{\partial E}{\partial y} = 2(y-y_1) + 2(y-y_2) + \cdots + 2(y-y_n) = 0$$

and $y = \dfrac{y_1 + y_2 + \cdots + y_n}{n}$

(12.6.6 on next page)

12.6.7 $f(\cancel{x_1, x_2) = x_1 x_2 + x_2}$ Typo. $f$ depends on $x_1, x_2$ and $x_3$

On another note, I wouldn't give you a 3x3 Hessian on the in-class portion of the exam. With these caveats,

$$\left.\begin{aligned} f_{x_1} &= x_2 + x_3 = 0 \\ f_{x_2} &= x_1 + x_3 = 0 \\ f_{x_3} &= x_2 + x_1 = 0 \end{aligned}\right\} \quad x_1 = x_2 = x_3 = 0$$

For the Hessian, $f_{x_1x_1} = 0$, $f_{x_1x_2} = 1$ $f_{x_1x_3} = 1$, and so on:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \Rightarrow \text{The eigenvalues are } \underbrace{-1, -1, 2}_{\text{mixed, so}}$$

the origin is a saddle

12.6.6

$f = x_1^3 - 3x_1x_2^2 + x_2^4$

$f_{x_1} = 3x_1^2 - 3x_2^2 = 3(x_1^2 - x_2^2)$

$f_{x_2} = -6x_1x_2 + 4x_2^3 = 2x_2(-3x_1 + 2x_2^2)$

For $f_{x_1} = 0$, $\quad x_1^2 = x_2^2$

For $f_{x_2} = 0$, $\quad x_2 = 0 \quad$ or $\quad -3x_1 + 2x_2^2 = 0$

$x_2 = 0 \downarrow x_1 = 0$

$-3x_1 + 2x_2^2 = 0 \ \downarrow\ x_1^2 = x_2^2$

$-3x_1 + 2x_1^2 = 0$

$x_1(-3 + 2x_1) = 0$

$x_1 = 0,\ x_2 = 0$

or $\quad x_1 = 3/2 \Rightarrow x_2 = \pm\frac{3}{2}$

So: $(0,0)$ or $(\frac{3}{2}, \pm 3/2)$ are the crit pts.

We examine the Hessian:

$$\begin{bmatrix} 6x_1 & -6x_2 \\ -6x_2 & 12x_2^2 - 6x_1 \end{bmatrix}$$

at $(0,0)$, $H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ so the test is inconclusive.

(ok to leave it there)

at $(3/2, 3/2)$,

$\begin{bmatrix} 9 & -9 \\ -9 & 18 \end{bmatrix}$ $\quad D = 81,\ f_{xx} > 0 \Rightarrow$ Local min

(same at $(3/2, -3/2)$)