Basic Problem (max)

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

Write the dual:

Basic Problem (max)

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

Write the dual:

$$
\begin{array}{llllll}
\min & w & -4y_1 & -12y_2 & -18y_3 & = 0 \\
\text{st} & & y_1 & & +3y_3 & \geq 3 \\
& & & y_2 & +2y_3 & \geq 5
\end{array}
$$

Basic Problem (max)

x_1	x_2	s_1	s_2	s_3	*rhs*
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

The current basis is:

Basic Problem (max)

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

The current basis is: $\mathcal{B} = \{s_1, s_2, s_3\}$
Given that, $\mathbf{c}_B^T$

Basic Problem (max)

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

The current basis is: $\mathcal{B} = \{s_1, s_2, s_3\}$
Given that, $\mathbf{c}_B^T = [0, 0, 0]$
so that the dual: $\mathbf{y}^T =$

Basic Problem (max)

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

The current basis is: $\mathcal{B} = \{s_1, s_2, s_3\}$
Given that, $\mathbf{c}_B^T = [0, 0, 0]$
so that the dual: $\mathbf{y}^T = \mathbf{c}_B^T B^{-1} = [0, 0, 0]$
And the excess variables: $-e_1 = 3 - y_1 - 3y_3$

Basic Problem (max)

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

The current basis is: $\mathcal{B} = \{s_1, s_2, s_3\}$
Given that, $\mathbf{c}_B^T = [0, 0, 0]$
so that the dual: $\mathbf{y}^T = \mathbf{c}_B^T B^{-1} = [0, 0, 0]$
And the excess variables: $-e_1 = 3 - y_1 - 3y_3 = 3$, or $e_1 = -3$
Similarly, $-e_2 = 5 - y_2 - 2y_3$

Basic Problem (max)

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

The current basis is: $\mathcal{B} = \{s_1, s_2, s_3\}$
Given that, $\mathbf{c}_B^T = [0, 0, 0]$
so that the dual: $\mathbf{y}^T = \mathbf{c}_B^T B^{-1} = [0, 0, 0]$
And the excess variables: $-e_1 = 3 - y_1 - 3y_3 = 3$, or $e_1 = -3$
Similarly, $-e_2 = 5 - y_2 - 2y_3$, or $e_2 = -5$.

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

This means that we have the following

x_1	x_2	s_1	s_2	s_3
0	0	4	12	18

e_1	e_2	y_1	y_2	y_3
-3	-5	0	0	0

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
3	2	0	0	1	18

This means that we have the following

x_1	x_2	s_1	s_2	s_3
0	0	4	12	18

e_1	e_2	y_1	y_2	y_3
-3	-5	0	0	0

The solution to the dual is FEASIBLE, the solution to the PRIMAL is INFEASIBLE.

x_1	x_2	s_1	s_2	s_3	rhs
-3	-5	0	0	0	0
1	0	1	0	0	4
0	2	0	1	0	12
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This means that we have the following

x_1	x_2	s_1	s_2	s_3
0	0	4	12	18

e_1	e_2	y_1	y_2	y_3
-3	-5	0	0	0

The solution to the dual is FEASIBLE, the solution to the PRIMAL is INFEASIBLE.

Note where the zeros appear in each solution...

Change basis $\mathcal{B} = \{s_1, s_2, x_1\}$, and recompute:

x_1	x_2	s_1	s_2	s_3	rhs
0	-3	0	0	1	18
0	$-2/3$	1	0	$-1/3$	-2
0	2	0	1	0	12
1	$2/3$	0	0	$1/3$	6

$$\mathbf{y}^T = [0, 0, 1]^T$$

Solve the primal and the dual:

x_1	x_2	s_1	s_2	s_3

Change basis $\mathcal{B} = \{s_1, s_2, x_1\}$, and recompute:

x_1	x_2	s_1	s_2	s_3	rhs
0	-3	0	0	1	18
0	$-2/3$	1	0	$-1/3$	-2
0	2	0	1	0	12
1	$2/3$	0	0	$1/3$	6

$$\mathbf{y}^T = [0, 0, 1]^T$$

Solve the primal and the dual:

x_1	x_2	s_1	s_2	s_3
6	0	-2	12	0

Change basis $\mathcal{B} = \{s_1, s_2, x_1\}$, and recompute:

x_1	x_2	s_1	s_2	s_3	rhs
0	-3	0	0	1	18
0	$-2/3$	1	0	$-1/3$	-2
0	2	0	1	0	12
1	$2/3$	0	0	$1/3$	6

$$\mathbf{y}^T = [0, 0, 1]^T$$

Solve the primal and the dual:

x_1	x_2	s_1	s_2	s_3
6	0	-2	12	0

e_1	e_2	y_1	y_2	y_3

Change basis $\mathcal{B} = \{s_1, s_2, x_1\}$, and recompute:

x_1	x_2	s_1	s_2	s_3	rhs
0	-3	0	0	1	18
0	$-2/3$	1	0	$-1/3$	-2
0	2	0	1	0	12
1	$2/3$	0	0	$1/3$	6

$$\mathbf{y}^T = [0, 0, 1]^T$$

Solve the primal and the dual:

x_1	x_2	s_1	s_2	s_3
6	0	-2	12	0

e_1	e_2	y_1	y_2	y_3
0	-3	0	0	1

Feasibility?

Change basis $\mathcal{B} = \{s_1, s_2, x_1\}$, and recompute:

x_1	x_2	s_1	s_2	s_3	rhs
0	-3	0	0	1	18
0	$-2/3$	1	0	$-1/3$	-2
0	2	0	1	0	12
1	$2/3$	0	0	$1/3$	6

$$\mathbf{y}^T = [0, 0, 1]^T$$

Solve the primal and the dual:

x_1	x_2	s_1	s_2	s_3
6	0	-2	12	0

e_1	e_2	y_1	y_2	y_3
0	-3	0	0	1

Feasibility?
The primal $\mathbf{x}$ is not feasible, the dual $\mathbf{y}$ is not feasible.

New basis: $\mathcal{B} = \{x_1, x_2, s_3\}$.

x_1	x_2	s_2	s_2	s_3	rhs
0	0	3	5/2	0	42
1	0	1	0	0	4
0	1	0	1/2	0	6
0	0	−3	−1	1	−6

$$\mathbf{y}^T = [3, 5/2, 0]^T$$

The solutions to primal and dual are:

x_1	x_2	s_1	s_2	s_3
4	6	0	0	−6

e_1	e_2	y_1	y_2	y_3
0	0	3	5/2	0

Feasibility?

New basis: $\mathcal{B} = \{x_1, x_2, s_3\}$.

x_1	x_2	s_2	s_2	s_3	rhs
0	0	3	5/2	0	42
1	0	1	0	0	4
0	1	0	1/2	0	6
0	0	−3	−1	1	−6

$$\mathbf{y}^T = [3, 5/2, 0]^T$$

The solutions to primal and dual are:

x_1	x_2	s_1	s_2	s_3
4	6	0	0	−6

e_1	e_2	y_1	y_2	y_3
0	0	3	5/2	0

Feasibility?
This is dual feasible, but primal infeasible.

Final basis: $\mathcal{B} = \{x_1, x_2, s_1\}$

x_1	x_2	s_2	s_2	s_3	rhs
0	0	0	$3/2$	1	36
1	0	0	$-1/3$	$1/3$	2
0	1	0	$1/2$	0	6
0	0	1	$1/3$	$-1/3$	2

$$\mathbf{y}^T = [0, 3/2, 1]^T$$

Solutions to the primal, dual are:

x_1	x_2	s_1	s_2	s_3
2	6	2	0	0

e_1	e_2	y_1	y_2	y_3
0	0	0	$3/2$	1

Feasibility?

Final basis: $\mathcal{B} = \{x_1, x_2, s_1\}$

x_1	x_2	s_2	s_2	s_3	rhs
0	0	0	$3/2$	1	36
1	0	0	$-1/3$	$1/3$	2
0	1	0	$1/2$	0	6
0	0	1	$1/3$	$-1/3$	2

$$\mathbf{y}^T = [0, 3/2, 1]^T$$

Solutions to the primal, dual are:

x_1	x_2	s_1	s_2	s_3
2	6	2	0	0

e_1	e_2	y_1	y_2	y_3
0	0	0	$3/2$	1

Feasibility?
Both the primal and dual are feasible (so optimal).