Review Questions, Final Exam

A few general questions

1. What does the Representation Theorem say (in linear programming)?
2. What is the Fundamental Theorem of Linear Programming?
3. What is the main idea behind the Simplex Method? (Think about what it is doing graphically- How does the algorithm start, how does it proceed?)

From Exam 1 Review

The numbering below is the same as that on the Exam 1 Review (Solutions online).

3. Convert the following LP to one in standard form. Write the result in matrix-vector form, giving $\mathbf{x}$, $\mathbf{c}$, A, $\mathbf{b}$ (from our formulation).

$$\begin{aligned} \min z = \quad & 3x - 4y + 2z \\ \text{st} \quad & 2x - 4y \geq 4 \\ & x + z \geq -5 \\ & y + z \leq 1 \\ & x + y + z = 3 \end{aligned}$$

   with $x \geq 0, y$ is URS, $z \geq 0$.

4. Consider again the "Wyndoor" company example we looked at in class:

$$\begin{aligned} \min z = \quad & 3x_1 + 5x_2 \\ \text{st} \quad & x_1 \leq 4 \\ & 2x_2 \leq 12 \\ & 3x_1 + 2x_2 \leq 18 \end{aligned}$$

   with x_1, x_2 both non-negative.

   (a) Rewrite so that it is in standard form.

   (b) Let s_1, s_2, s_3 be the extra variables introduced in the last answer. Is the following a basic solution? Is it a basic feasible solution?

$$x_1 = 0, x_2 = 6, s_1 = 4, s_2 = 0, s_3 = 6$$

   Which variables are BV, and which are NBV?

   (c) Find the basic feasible solution obtained by taking s_1, s_3 as the non-basic variables.

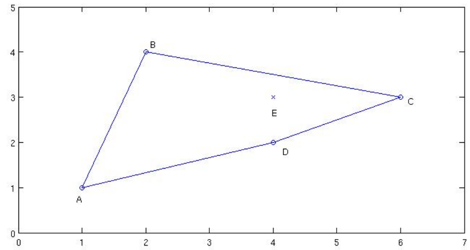


Figure 1: Figure for the convex combinations, Exercise 5.

5. Consider Figure 1, with points $A(1,1), B(1,4)$ and $C(6,3)$, $D(4,2)$ and $E(4,3)$.
   - Write the point E as a convex combination of points A, B and C.
   - Can E be written as a convex combination of A, B and D? If so, construct it.
   - Can A be written as a *linear* combination of A, B and D? If so, construct it.

13. Let a feasible region be defined by the system of inequalities below:

$$\begin{aligned} -x_1 + 2x_2 &\leq 6 \\ -x_1 + x_2 &\leq 2 \\ x_2 &\geq 1 \\ x_1, x_2 \geq 0 & \end{aligned}$$

The point $(4,3)$ is in the feasible region. Find vectors $\mathbf{d}$ and $\mathbf{b}_1, \cdots \mathbf{b}_k$ and constants σ_i so that the Representation Theorem is satisfied (NOTE: Your vector $\mathbf{x}$ from that theorem is more than two dimensional).

14. Let a feasible region be defined by the system of inequalities below:

$$\begin{aligned} -x_1 + x_2 &\leq 2 \\ x_1 - x_2 &\leq 1 \\ x_1 + x_2 &\leq 5 \\ x_1, x_2 \geq 0 & \end{aligned}$$

The point $(2,2)$ is in the feasible region. Find vectors $\mathbf{d}$ and $\mathbf{b}_1, \cdots \mathbf{b}_k$ and constants σ_i so that the Representation Theorem is satisfied (NOTE: Your vector $\mathbf{x}$ from that theorem is more than two dimensional).

18. Given the current tableau (with variables labeled above the respective columns), answer the questions below.

x_1	x_2	s_1	s_2	rhs
0	-1	0	2	24
0	$1/3$	1	$-1/3$	1
1	$2/3$	0	$1/3$	4

(a) Is the tableau optimal (and did your answer depend on whether we are maximizing or minimizing)? For the remaining questions, you may assume we are maximizing.

(b) Give the current BFS.

(c) Directly from the tableau, can I increase x_2 from 0 to 1 and remain feasible? Can I increase it to 4?

(d) If x_2 is increased from 0 to 1, compute the new value of z, x_1, s_1 (assuming s_2 stays zero).

(e) Write the objective function and all variables in terms of the non-basic (or free) variables, and then put them in vector form.

From the Chapter 4 Review

1. Use the simplex algorithm to find *two* optimal solutions to the following LP:

$$\begin{aligned} \max z = &\ 5x_1 + 3x_2 + x_3 \\ \text{st} &\ x_1 + x_2 + 3x_3 \le 6 \\ &\ 5x_1 + 3x_2 + 6x_3 \le 15 \end{aligned}$$

with x_1, x_2 both non-negative.

2. Use the simplex algorithm to find the optimal solution to the following LP:

$$\begin{aligned} \max z = &\ -4x_1 + x_2 \\ \text{st} &\ 3x_1 + x_2 \le 6 \\ &\ -x_1 + 2x_2 \le 0 \end{aligned}$$

with x_1, x_2 both non-negative.

3. Use the big-M and the two-phase method to find the optimal solution to the following LP:

$$\begin{aligned} \max z = &\ 5x_1 - x_2 \\ \text{st} &\ 2x_1 + x_2 = 6 \\ &\ x_1 + x_2 \le 4 \\ &\ x_1 + 2x_2 \le 5 \end{aligned}$$

with x_1, x_2 both non-negative.

7. Use the simplex algorihtm to find two optimal solutions to the following LP:

$$\begin{aligned} \max z = &\ 4x_1 + x_2 \\ \text{st} &\ 2x_1 + 3x_2 \le 4 \\ &\ x_1 + x_2 \le 1 \\ &\ 4x_1 + x_2 \le 2 \end{aligned}$$

with x_1, x_2 both non-negative.

15. Consider a maximization problem with the optimal tableau given below. First, give the optimal solution. Next, determine the second-best solution (think about the sets of variables that could have come before the current set).

x_1	x_2	x_3	x_4	rhs
2	1	0	0	10
3	2	1	0	3
4	3	0	1	5

17. You are given the tableau below for a maximization problem. Give conditions on a_1, a_2, a_3, b, c that make the following statements true:

(a) The current solution is optimal.

(b) The current solution is optimal, and there are alternate optimal solutions.

(c) The LP is unbounded (assume $b \geq 0$ for this part).

x_1	x_2	x_3	x_4	x_5	rhs
$-c$	2	0	0	0	10
-1	a_1	1	0	0	4
a_2	-4	0	1	0	1
a_3	3	0	0	1	b

18. Suppose we have obtained the tableau below for a maximization problem. State conditions on $a_1, a_2, a_3, b, c_1, c_2$ that are required to make the following statements true:

(a) The current solution is optimal, and there are alternative optimal solutions.

(b) The current basic solution is not a BFS.

(c) The current basic solution is a degenerate BFS.

(d) The current basic solution is feasible, but the LP is unbounded.

(e) The current basic solution is feasible, but the objective function can be improved by replacing x_6 with x_1 as a basic variable.

x_1	x_2	x_3	x_4	x_5	x_6	rhs
c_1	c_2	0	0	0	0	10
4	a_1	1	0	a_2	0	b
-1	-5	0	1	-1	0	2
a_3	-3	0	0	-4	1	3

From the Group Work Problem
(Exam 2 Review Links on the Class Website)

In the textbook's "Dakota Problem", we are making desks, tables and chairs, and we want to maximize profit given constraints on lumber, finishing and carpentry (resp).

For the primal, let x_1, x_2, x_3 be the number of desks, tables and chairs we make (resp), where the original (max) tableau is as given below:

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & rhs \\ -60 & -30 & -20 & 0 & 0 & 0 & 0 \\ \hline 8 & 6 & 1 & 1 & 0 & 0 & 48 \\ 4 & 2 & \frac{3}{2} & 0 & 1 & 0 & 20 \\ 2 & \frac{3}{2} & \frac{1}{2} & 0 & 0 & 1 & 8 \end{array} \quad \rightarrow \quad \begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & rhs \\ 0 & 5 & 0 & 0 & 10 & 10 & 280 \\ \hline 0 & -2 & 0 & 1 & 2 & -8 & 24 \\ 0 & -2 & 1 & 0 & 2 & -4 & 8 \\ 1 & \frac{5}{4} & 0 & 0 & -\frac{1}{2} & \frac{3}{2} & 2 \end{array}$$

1. Write the down vectors/matrices that we typically use in our computations. Namely, $\mathbf{c}, \mathbf{c}_B, B$, and B^{-1}.

2. Using our vector notation, if $\mathcal{B}$ gives the optimal basis, how do we compute the dual, $\mathbf{y} =?$

3. Write down the dual (either as an initial tableau or in "normal form").

4. Using the optimal Row 0 from the primal, write down the solution to the dual:

5. In our "normal form", we have $A\mathbf{x} \leq \mathbf{b}$ for the primal and $A^T\mathbf{y} \geq \mathbf{c}$ for the dual. We will define two "slacks"[1]

   - The "slack" for the primal, given $\mathbf{x}$: $\mathbf{b} - A\mathbf{x}$. Compute the current slack for the primal.
   - The "slack" for the dual, given $\mathbf{y}$: $A^T\mathbf{y} - \mathbf{c}$. Compute the current slack for the dual. You can use your answer to (3) if necessary.

6. What is the shadow price for each constraint?

7. Write down the inequalit(ies) we need for Δ if we change the coefficient of x_2 from 30 to $30 + \Delta$, and we want the current basis to remain optimal.

8. Write down the inequalit(ies) we need for Δ if we change the coefficient of x_3 from 20 to $20 + \Delta$, and we want the current basis to remain optimal.

9. How does changing a *column* of A effect the dual? Use this to see what would happen if we change the column for x_2 (tables) to be $[5, 2, 2]^T$- Is it now worth it to make tables?

[1] Sorry, the vocabulary is related to the "slack variable", but we're using "slack" in a different context now. Ask if you're ever not sure which we're talking about.

10. How does creating a new column of A effect the dual? Use this to see if it makes sense to manufacture footstools, where we sell them for \$15 each, and the resources are $[1,1,1]^T$.

From the Exam 2 Review

6. Consider the LP and the optimal tableau with missing Row 0 and missing optimal RHS (assume big-M).

$$\begin{aligned} \max z = \; & 3x_1 + x_2 \\ \text{s.t.} \; & 2x_1 + x_2 \le 4 \\ & 3x_1 + 2x_2 \ge 6 \\ & 4x_1 + 2x_2 = 7 \\ & x_1, x_2 \ge 0 \end{aligned}$$

x_1	x_2	s_1	e_2	a_2	a_3	rhs
0	0	1	0	0	$-1/2$	
0	1	0	-2	2	$-3/2$	
1	0	0	1	-1	1	

Find Row 0 and the RHS for the optimal tableau (without performing row reductions!)

7. Give an argument why, if the primal is unbounded, then the dual must be infeasible.

8. In solving the following LP, we obtain the optimal tableau shown:

$$\begin{aligned} \max z = \; & 6x_1 + x_2 \\ \text{st} \; & x_1 + x_2 \le 5 \\ & 2x_1 + x_2 \le 6 \\ & x_1, x_2 \ge 0 \end{aligned} \quad \Rightarrow$$

x_1	x_2	s_1	s_2	rhs
0	2	0	3	18
0	1/2	1	$-1/2$	2
1	1/2	0	1/2	3

(a) If we add a new constraint, is it possible that we can increase z? Why or why not?

(b) If we add the constraint $3x_1 + x_2 \le 10$, is the current basis still optimal?

(c) If we add the constraint $x_1 - x_2 \ge 6$, we can quickly see that the optimal solution changes. Find out if we have a new optimal solution or if we have made the problem infeasible.

(d) Same question as the last one, but let's change the constraint to $8x_1 + x_2 \le 12$.

(e) If I add a new variable x_3 so that:

$$\begin{aligned} \max z = \; & 6x_1 + x_2 + x_3 \\ \text{st} \; & x_1 + x_2 + 2x_3 \le 5 \\ & 2x_1 + x_2 + x_3 \le 6 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Does the current basis stay optimal? Answer two ways- One using the optimal tableau, and the second using the dual.

From the Chapter 6 Review

1. Consider the following LP and its optimal tableau, shown.

$$\begin{aligned} \max z = \;& 4x_1 + x_2 \\ \text{st}\;\; & x_1 + 2x_2 = 6 \\ & x_1 - x_2 \geq 3 \\ & 2x_1 + x_2 \leq 10 \\ & x_1, x_2 \geq 0 \end{aligned} \quad \Rightarrow \quad \begin{array}{cccccc|c} x_1 & x_2 & e_1 & s_1 & a_1 & a_2 & rhs \\ 0 & 0 & 0 & 7/3 & M-2/3 & M & 58/3 \\ \hline 0 & 1 & 0 & -1/3 & 2/3 & 0 & 2/3 \\ 1 & 0 & 0 & 2/3 & -1/3 & 0 & 14/3 \\ 0 & 0 & 1 & 1 & -1 & -1 & 1 \end{array}$$

   (a) Find the dual of this LP and its optimal solution.

   (b) Find the range of values of $b_3 = 10$ for which the current basis remains optimal.

3. Consider the following LP and its optimal tableau, shown.

$$\begin{aligned} \max z = \;& 5x_1 + x_2 + 2x_3 \\ \text{st}\;\; & x_1 + x_2 + x_3 \leq 6 \\ & 6x_1 + x_3 \leq 8 \\ & x_2 + x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \quad \Rightarrow \quad \begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & rhs \\ 0 & 1/6 & 0 & 0 & 5/6 & 7/6 & 9 \\ \hline 0 & 1/6 & 0 & 1 & -1/6 & -5/6 & 3 \\ 1 & -1/6 & 0 & 0 & 1/6 & -1/6 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 2 \end{array}$$

   (a) Find the dual of this LP and its optimal solution.

   (b) Find the range of values of $c_1 = 5$ for which the current basis remains optimal.

   (c) Find the range of values of $c_2 = 1$ for which the current basis remains optimal.

6. Consider the following LP and its optimal tableau, shown.

$$\begin{aligned} \max z = \;& 3x_1 + x_2 - x_3 \\ \text{st}\;\; & 2x_1 + x_2 + x_3 \leq 8 \\ & 4x_1 + x_2 - x_3 \leq 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \quad \Rightarrow \quad \begin{array}{ccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & rhs \\ 0 & 0 & 1 & 1/2 & 1/2 & 9 \\ \hline 0 & 1 & 3 & 2 & -1 & 6 \\ 1 & 0 & -1 & -1/2 & 1/2 & 1 \end{array}$$

   (a) Find the dual of this LP and its optimal solution.

   (b) Find the range of values of $b_2 = 1$ for which the current basis remains optimal. If $b_2 = 12$, what is the new optimal solution?

9. Consider the following LP and its optimal tableau, shown.

$$\begin{aligned} \max z = \;& 3x_1 + 4x_2 + x_3 \\ \text{st}\;\; & x_1 + x_2 + x_3 \leq 50 \\ & 2x_1 - x_2 + x_3 \geq 15 \\ & x_1 + x_2 = 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \quad \Rightarrow \quad \begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & e_1 & a_1 & a_2 & rhs \\ 1 & 0 & 0 & 1 & 0 & M & M+3 & 80 \\ \hline -3 & 0 & 0 & 1 & 1 & -1 & -2 & 15 \\ 0 & 0 & 1 & 1 & 0 & 0 & -2 & 40 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 10 \end{array}$$

(a) Find the dual of this LP and its optimal solution.

(b) Find the range of values of $b_2 = 15$ for which the current basis remains optimal. If $b_2 = 5$, what is the new optimal solution?

10. Consider the following LP and its optimal tableau, shown.

$$\begin{aligned} \max z = \;& 3x_1 + 2x_2 \\ \text{st} \;& 2x_1 + 5x_2 \leq 8 \\ & 3x_1 + 7x_2 \leq 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \quad \Rightarrow \quad \begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & rhs \\ 0 & 0 & 0 & 1 & 10 \\ \hline 0 & 1/3 & 1 & -2/3 & 4/3 \\ 1 & 7/3 & 0 & 1/3 & 10/3 \end{array}$$

(a) Find the dual of this LP and its optimal solution.

(b) Find the range of values of $b_2 = 15$ for which the current basis remains optimal. If $b_2 = 5$, what is the new optimal solution?

18. Consider the following LP and its optimal tableau, shown.

$$\begin{aligned} \max z = \;& -4x_1 - x_2 \\ \text{st} \;& 4x_1 + 3x_2 \geq 6 \\ & x_1 + 2x_2 \leq 3 \\ & 3x_1 + x_2 = 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \quad \Rightarrow \quad \begin{array}{cccccc|c} x_1 & x_2 & e_1 & s_1 & a_1 & a_2 & rhs \\ 0 & 0 & 0 & 1/5 & M & M - \frac{7}{5} & -18/5 \\ \hline 0 & 1 & 0 & 3/5 & 0 & -1/5 & 6/5 \\ 1 & 0 & 0 & -1/5 & 0 & 2/5 & 3/5 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 \end{array}$$

(a) Find the dual of this LP and its optimal solution.

(b) If we changed the LP to:

$$\begin{aligned} \max z = \;& -4x_1 - x_2 - x_3 \\ \text{st} \;& 4x_1 + 3x_2 + x_3 \geq 6 \\ & x_1 + 2x_2 + x_3 \leq 3 \\ & 3x_1 + x_2 + x_3 = 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

would the current solution remain optimal?

20. Use the Theorem of Complementary Slackness to find the optimal solution to the following LP and its dual. To assist you, note that $y_1 = y_2 = 1$ is a solution to the dual.

$$\begin{aligned} \max z = \;& 3x_1 + 4x_2 + x_3 + 5x_4 \\ \text{st} \;& x_1 + 2x_2 + x_3 + 2x_4 \leq 5 \\ & 2x_1 + 3x_2 + x_3 + 3x_4 \leq 8 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

33. Consider the following LP:

$$\begin{aligned} \max z = \ & c_1x_1 + c_2x_2 \\ \text{st} \ & 3x_1 + 4x_2 \le 6 \\ & 2x_1 + 3x_2 \le 4 \\ & x_1, x_2 \ge 0 \end{aligned} \quad \Rightarrow \quad \begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & rhs \\ 0 & 0 & 1 & 2 & 14 \\ \hline 1 & 0 & 3 & -4 & 2 \\ 0 & 1 & -2 & 3 & 0 \end{array}$$

Do as little work as possible to determine c_1, c_2.

34. Consider the following LP and its partially determined optimal tableau below (missing values are denoted by ?).

$$\begin{aligned} \max z = \ & 20x_1 + 10x_2 \\ \text{st} \ & x_1 + x_2 = 150 \\ & x_1 \le 40 \\ & x_2 \ge 20 \\ & x_1 \ge 0 \end{aligned} \quad \Rightarrow \quad \begin{array}{cccccc|c} x_1 & x_2 & a_1 & s_1 & e_1 & a_2 & rhs \\ 0 & 0 & ? & ? & 0 & ? & 1900 \\ \hline 0 & 0 & 1 & -1 & 1 & -1 & 90 \\ 1 & 0 & 0 & 1 & 0 & 0 & 40 \\ 0 & 1 & 1 & -1 & 0 & 0 & 110 \end{array}$$

(a) Compute the optimal tableau.

(b) Find the dual and the optimal solution to the dual.

From the Chapter 7 Review

1. Televco produces TV tubes at three plants, shown below. We have three customers, and the profits for each depend on the plant, as shown on the right.

Plant	1	2	3
Number of Tubes	50	100	50

	Cust 1	Cust 2	Cust 3
Plant 1	75	60	69
Plant 2	79	73	68
Plant 3	85	76	70

- Formulate a balanced transportation porblem that can be used to maximize profits.
- Use the NW corner method to find a BFS to the problem.
- Find an optimal solution.

2. Five workers are available to perform four jobs. The time it takes each worker to perform each job is given below. The goal is to assign workers to jobs so as to minimize

the total time required. Use the Hungarian method to solve.

	Job 1	2	3	4
Worker 1	10	15	10	15
2	12	8	20	16
3	12	9	12	18
4	6	12	15	18
5	16	12	8	12

3. A company must meet the demands shown below for a product. Demand may be backlogged at a cost of \$5 per unit per month. All demand must be met at the end of March. Thus, if 1 unit of January demand is met during March, a cost of \$5$\times$ 2=\$10 is incurred. Monthly production capacity and unit production cost during each month are shown below. A holding cost of \$20 per unit is assessed on the inventory at the end of each month.

Month	Demand	Prod Cap	Unit Prod Cost
Jan	30	35	400
Feb	30	30	420
Mar	20	35	410

- Formulate a balanced transportation problem that can be used to determine how to minimize the cost (including backlogging, holding and production costs).
- Use Vogel's method (VAM) to find a basic feasible solution.
- Find the optimal solution.

HINT: To set this up, think of Jan, Feb and Mar as having supplies as $35, 30 and 35$, and demands as $30, 30, 20$ (we'll need a dummy to balance. For the costs, January can supply January at a cost of \$400 per unit, January can supply Feb at a cost of \$420 per unit, and it can supply March at a cost of \$440 per unit.

4. Appletree cleaning has five cleaning people. To complete cleaning the house, they must vacuum, clean the kitchen, clean the bathroom, and do general straightening up. The time it takes each person to do each job is shown below. If each person can be assigned at most one job, set up and solve the table for the assignment problem using

the Hungarian method.

	Job 1	2	3	4
Worker 1	6	5	2	1
2	9	8	7	3
3	8	5	9	4
4	7	7	8	3
5	5	5	6	4

8. Use the NW corner method to find a BFS, then solve the unbalanced (minimization) problem shown below:

12	14	16	60
14	13	19	50
17	15	18	40
40	70	10	

9. Solve the following LP (HINT: It can be put into a 2×2 transportation problem).

$$\begin{aligned} \min z = \ & 2x_1 + 3x_2 + 4x_3 + 3x + 4 \\ s.t. \ & x_1 + x_2 \leq 4 \\ & x_3 + x_4 \leq 5 \\ & x_1 + x_3 \geq 3 \\ & x_2 + x_4 \geq 6 \end{aligned}$$

(All variables ≥ 0).

10. Find the optimal solution to the balanced transportation problem below:

4	2	4	15
12	8	4	15
10	10	10	

For the next two problems, we'll need the optimal tableau for the PowerCo problem (p 391):

8	6 10	10 25	9	35
9 45	12	13 5	7	50
14	9 10	16	5 30	40
45	20	30	30	

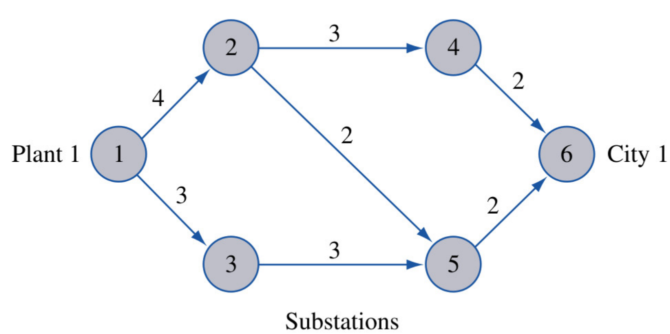
15. For the PowerCo problem, find the range of values of c_{24} for which the current basis is optimal.

16. For the PowerCo problem, find the range of values of c_{23} for which the current basis is optimal.

From Chapter 8

The review for this chapter was pretty terrible, so here are some review questions.

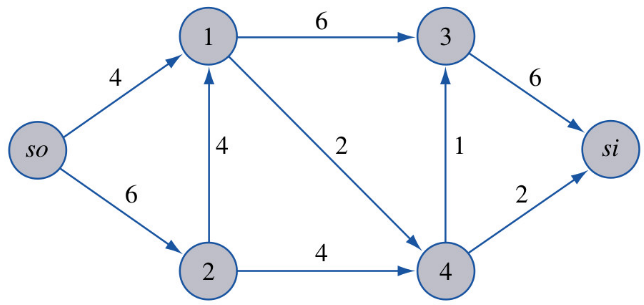
1. Write the shortest path problem as (i) a transhipment problem, and (ii) a linear program. For specificity, use the PowerCo network below (Figure 2, p 414). (Hints: For transhipment, we have one supply, one demand, and a bunch of warehouses. For the LP, you could write it from the transhipment problem.). Finally, find the shortest path from Plant 1 to City 1 using Dijkstra's algorithm.



2. Given the figure below (Fig 23 from the text), first write the maximum flow problem as a linear program. (Hint: Think about the constraints on the flow for each edge,

then for each vertex). Solve the max-flow problem using Ford-Fulkerson. Be sure to write out the residual graphs. Finally, find a cut giving the minimum capacity to show that your solution is correct.

FIGURE 23



3. Continuing with Figure 23 from the previous question, with the maximum flow, if the cut is:

$$A = \{so, 2, 3\}, B = \{1, 4, si\}$$

then what is the net flow across the cut? What is the capacity of the cut?

4. To use a max-flow for the assignment problem, recall that we have a source node that goes to a node for each person (capacity of 1 each), then each person is attached to a job (capacity of 1 for each edge), and then each job is attached to a sink. (NOTE: We are maximizing the number of compatible pairs.)

(Problem 7, 8.3) Four workers are available to perform four jobs, but not all workers may be assigned to every job (See the chart below, X marks compatible). Draw the network for the maximum flow problem that can be used to determine whether all jobs can be assigned to a suitable worker.

	Job 1	Job 2	Job 3	Job 4
Worker 1	X	–	–	–
2	X	X	–	–
3	–	X	–	–
4	X	X	X	X

5. (Figure 4 from text below) (a) If we look at the edges as having costs, find the path from node 1 to all other nodes. (b) If we look at the edges as capacities for a flow, find the maximum flow.

FIGURE 4

Network for Problem 2