A Worked Example, Big M

$$\begin{array}{rllll} \min z = & 4x_1 & +x_2 & & \\ \text{st} & 3x_1 & +x_2 & = 3 & \\ & 4x_1 & +3x_2 & \geq 6 & \\ & x_1 & +2x_2 & \leq 4 & \end{array} \quad \Rightarrow$$

x_1	x_2	e_1	s_1	a_1	a_2	
4	1	0	0	M	M	0
3	1	0	0	1	0	3
4	3	-1	0	0	1	6
1	2	0	1	0	0	4

(all variables non-negative)

```
with(LinearAlgebra):
A:=<<4,3,4,1>|<1,1,3,2>|<0,0,-1,0>|<0,0,0,1>|<M,1,0,0>|<M,0,1,0>|<0,3,6,4>>;
```

1. Step 1: Clean up Row 0.

```
A1:=RowOperation(A,[1,2],-M):
A2:=RowOperation(A1,[1,3],-M);
```

$4-7M$	$1-4M$	M	0	0	0	$-9M$
3	1	0	0	1	0	3
4	3	-1	0	0	1	6
1	2	0	1	0	0	4

2. Standard simplex: Pivot in first column, first constraint.

```
A3:=RowOperation(A2,2,1/3):
A4:=RowOperation(A3,[1,2],-4+7*M):
A5:=RowOperation(A4,[3,2],-4):
A6:=RowOperation(A5,[4,2],-1);
```

0	$\frac{1}{3}-\frac{5}{3}M$	M	0	$-\frac{4}{3}+\frac{7}{3}M$	0	$-4-2M$
1	$1/3$	0	0	$1/3$	0	1
0	$5/3$	-1	0	$-4/3$	1	2
0	$5/3$	0	1	$-1/3$	0	3

3. For the Ratio test, `evalf([1/(1/3), 2/(5/3),3/(5/3)]);`.

   Pivot in constraint 2 (Row 3):

```
A7:=RowOperation(A6,3,3/5):
A8:=RowOperation(A7,[1,3],1/3+(5/3)*M):
A9:=RowOperation(A8,[2,3],-1/3):
A10:=RowOperation(A9,[4,3],-5/3);
```

0	0	$-1/5$	0	$-\frac{8}{5}+M$	$\frac{1}{5}+M$	$-18/5$
1	0	$1/5$	0	$3/5$	$-1/5$	$3/5$
0	1	$-3/5$	0	$-4/5$	$3/5$	$6/5$
0	0	1	1	1	-1	1

4. STOP: Notice that at this stage, we have a feasible solution to the original problem (it is not optimal yet). we can either remove or ignore the artificial variables. To continue as usual, pivot in the third column, third constraint (Row 4):

```
A11:=RowOperation(A10,[1,4],1/5):
A12:=RowOperation(A11,[2,4],-1/5):
A13:=RowOperation(A12,[3,4],3/5);
```

0	0	0	$1/5$	$*$	$*$	$-17/5$
1	0	0	$-1/5$	$*$	$*$	$2/5$
0	1	0	$3/5$	$*$	$*$	$9/5$
0	0	1	1	$*$	$*$	1

Examples, Big M Method

1. Here is the LP:

$$\begin{array}{rlll} \max z = & -x_1 & +2x_2 & \\ \text{st} & x_1 & +x_2 & \geq 2 \\ & -x_1 & +x_2 & \geq 1 \\ & & x_2 & \leq 3 \end{array}$$

   Initial tableau using Big-M

	x_1	x_2	e_1	e_2	s_1	a_1	a_2	
	1	-2	0	0	0	M	M	0
a_1	1	1	-1	0	0	1	0	2
a_2	-1	1	0	-1	0	0	1	1
s_1	0	1	0	0	1	0	0	3

   Write the final tableau and the interpretation:

2. This is the LP:

$$\begin{array}{lrrrl} \max z = & x_1 & +3x_2 & +x_3 & \\ \text{st} & x_1 & +x_2 & +2x_3 & \leq 4 \\ & -x_1 & & +x_3 & \geq 4 \\ & & & x_3 & \geq 3 \end{array}$$

Write the initial and final tableau and the interpretation:

3. Here is the LP

$$\begin{array}{lrrrrl} \max z = & x_1 & +x_2 & & & \\ \text{st} & x_1 & -x_2 & -x_3 & & = 1 \\ & -x_1 & +x_2 & +2x_3 & -x_4 & = 1 \end{array}$$

Write the initial and final tableau and the interpretation: