Sensitivity Analysis and LINGO

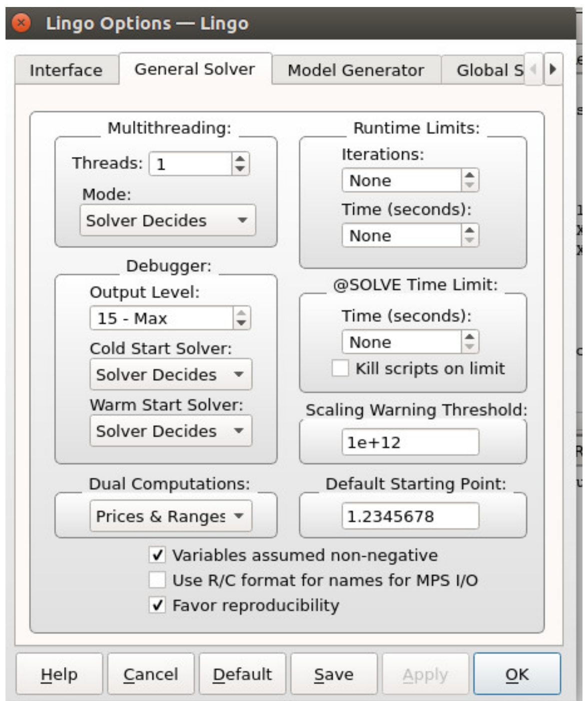
Before we start, we need to set an option in LINGO. After that, we'll proceed with a simple example.

Setting Options for Ranges in LINGO

There is a particular tab that needs to be changed in order to get a range report in LINGO.

Under *Solver*, go to *Options*. Find a tab at the top of the dialog box that says *General Solver*, then find the box titled **Dual Computations**. Set that box so compute **Prices and Ranges** (the default is prices only).

You should select *Apply* and *Save* (so that you don't have to set the options every time). Here's the dialog box from the Linux version:



Example

Here is an LP that we could solve using a graphical technique (It is exercise 1, section 3.1, where x_1 is the acres of corn, x_2 is the acres of wheat):

$$\begin{array}{llrrll}
\max z & = & 30x_1 & +100x_2 & & \\
\text{s.t.} & & x_1 & +x_2 & \leq 7 & \text{(Land Constraint)} \\
 & & 4x_1 & +10x_2 & \leq 40 & \text{(Labor Constraint)} \\
 & & 10x_1 & & \geq 30 & \text{(Govt. Constraint)} \\
 & & x_1 \geq 0 & x_2 \geq 0 & &
\end{array}$$

We input the linear program into the "Lingo Model" window:

```
max=30*x1+100*x2;
x1+x2<=7;
4*x1+10*x2<=40;
10*x1>=30;
```

Now solve the model using either the *Solver* tab or the dartboard icon.

We get the following output in the "Solution Report":

```
Global optimal solution found.
  Objective value:                              370.0000

...

 Variable           Value        Reduced Cost
       X1        3.000000            0.000000
       X2        2.800000            0.000000

      Row    Slack or Surplus      Dual Price
        1        370.0000            1.000000
        2        1.200000            0.000000
        3        0.000000            10.00000
        4        0.000000           -1.000000
```

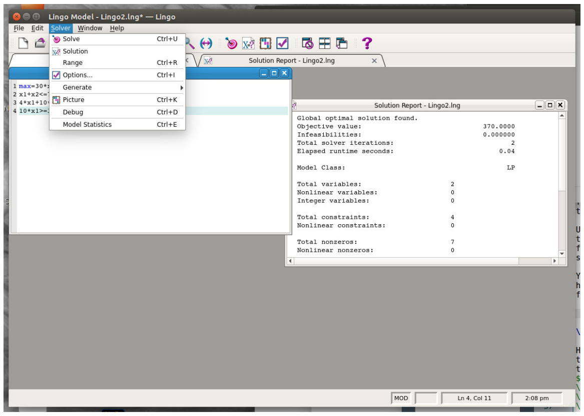
We haven't discusses "Reduced Cost" or "Dual Price" yet- Let's try an experiment. If you increase the RHS of constraint 2 so that:

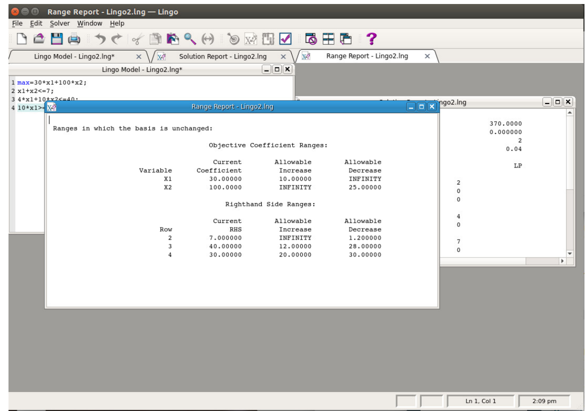
$$4x_1 + 10x_2 \leq 41;$$

How does that change z? If you solve it, $z = 380$ now- An increase of 10. This is the *shadow price*, or *dual price*. We notice that the shadow price of the first constraint (Row 2) is zero- That's because, since that constraint isn't currently binding, it doesn't matter if we increase the RHS. On the other hand, increasing the RHS of the last constraint by 1 unit will decrease the value of z by 1.

NOTE: Our text says that the **shadow price** is the amount by which z is *improved* when a constraint (RHS) increases by 1. Careful here- For the maximization problem, positive shadow prices increase the value of z, but for a minimization problem, a positive shadow price will *decrease* the value of z.

To see the range for each variable, we have to click (or select) the model window, then select *Range* under the *Solver* tab as shown below, which should result in the range report (right image, below).





In this case, we get the following report:

```
Ranges in which the basis is unchanged:

                          Objective Coefficient Ranges:

                    Current        Allowable      Allowable
Variable        Coefficient         Increase       Decrease
      X1           30.00000         10.00000       INFINITY
      X2           100.0000         INFINITY       25.00000

                              Righthand Side Ranges:

                    Current        Allowable      Allowable
     Row                RHS         Increase       Decrease
       2           7.000000         INFINITY       1.200000
       3           40.00000         12.00000       28.00000
       4           30.00000         20.00000       30.00000
```

Note that these are the allowable *changes*, not the actual values, by which we can change the current values (of the coefficients shown or the RHS of the constraints) and maintain the current set of basic variables.

For example, the 30 for x_1 can be increased to 40, and we would still have the same set of basic variables comprise the BFS at which the optimum occurs.

By the way, we assume these changes are done *one at a time.* There is an analysis one can perform for multiple changes, but we will not look at that this semester (Section 6.4).