Summary of Section 7.1

- Be able to define a (balanced) transportation problem.
- Be able to construct a transportation tableau.
- Look at possible solutions to a transportation problem.
- Be able to use Excel to solve a basic transportation problem.

Transportation Problems, Lecture 1

Here's an example of a transportation problem:

A company sends goods from two warehouses to three customers. Warehouse 1 has 50 units available, warehouse 2 has 40. Each customer wants 30 units. The shipping costs are given below. Formulate an LP to minimize costs to get goods to the customers.

	$C1$	$C2$	$C3$	Avail
$W1$	15	35	25	50
$W2$	10	50	40	40
Demand	30	30	30	

It's straightforward to convert this to an LP. Let x_{ij} be the number of units from warehouse i to customer j. Then we have 2×3 variables. Using our table, we can construct the LP:

$$\min\ z = 15x_{11} + 35x_{12} + 25x_{13} + 10x_{21} + 50x_{22} + 40x_{23}$$

s.t.

$$\begin{aligned} x_{11} + x_{12} + x_{13} &\leq 50 \text{ Outgoing from Warehouse 1} \\ x_{21} + x_{22} + x_{23} &\leq 40 \text{ Outgoing from Warehouse 2} \\ x_{11} + x_{21} &\geq 30 \text{ Incoming to Customer 1} \\ x_{12} + x_{22} &\geq 30 \text{ Incoming to Customer 2} \\ x_{13} + x_{23} &\geq 30 \text{ Incoming to Customer 3} \end{aligned}$$

Because of the structure of this problem, we can specialize the tableau to something called a **transportation tableau**.

	Cust 1	Cust 2	Cust 3	Supply
WH 1	15	35	25	50
WH 2	10	50	40	40
Demand	30	30	30	?

Before we give algorithms to solve this, try getting a solution on your own (try to go as cheaply as possible!)

The General Transportation Problem

This is a specific case of a more general transportation problem:

	Sink 1	Sink 2	$\cdots$	Sink n	
Source 1	c_{11}	c_{12}	$\cdots$	c_{1n}	s_1
Source 2	c_{21}	c_{22}	$\cdots$	c_{2n}	s_2
$\vdots$					$\vdots$
Source m	c_{m1}	c_{m2}	$\cdots$	c_{mn}	s_m
Demand	d_1	d_2	$\cdots$	d_n	$\sum s_i = \sum d_i$

- We have m supply points and n demand points.
- In the standard problem, each supply point can distribute to any demand point (in particular, a supply point is not moving goods to another supply point).
- Each supply point has a certain volume available (s_i).
- Each demand point needs a certain volume (d_i).
- There is a specific cost moving from i to j: c_{ij}
- The amount of material moving from i to j is typically x_{ij}.

Remember that this is just a summary of the underlying linear program:

$$\min\ z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

such that:

- Each supply constraint is met. That is, the sum of all product going out of supply point i should not exceed what is available. This is the sum from left to right in the table.

$$\sum_{j=1}^{n} x_{ij} \le s_i$$

- Each demand constraint is met. That is, the sum of all incoming product to demand point j should meet or exceed what we need. This is the sum from top to bottom in the table.

$$\sum_{i=1}^{m} x_{ij} \geq d_j$$

- All x_{ij} should be non-negative.

Questions for the class

- How many variables do we have? (mn)
- How many equations do we have? $(m+n)$
- If the following holds, do we have a solution?

$$\sum_{i=1}^{m} s_i < \sum_{j=1}^{n} d_j$$

Definition:

If the supply equals the demand for a given transportation problem, then the problem is said to be **balanced**. The techniques we develop will be for **balanced** problems.

If the problem is not balanced, it is possible to introduce *dummy* or *virtual* supply or demand points to make the problem balanced. We'll look at this later.

Setting up a problem that is not balanced

If the problem is not balanced, we may be able to introduce either a dummy warehouse or dummy customer so that the new problem is balanced.

(Text example) Suppose we have two reservoirs of water supplying 3 cities. Each reservoir can supply 50 million gallons, and each city wants 40 million gallons. There is a penalty for not meeting the demand for each city: \$20, \$22, \$23 respectively. The costs are given in the table below. *Formulate this LP as a balanced transportation problem.*

	City 1	City 2	City 3	Avail
Res 1	7	8	10	50
Res 2	9	7	8	50
Demand	40	40	40	

SOLUTION:

	City 1	City 2	City 3	Avail
Res 1	7	8	10	50
Res 2	9	7	8	50
Dummy	20	22	23	20
Demand	40	40	40	120

Summary: Balanced Transportation Problem and Tableau

Now, we define a balanced transportation problem as:

$$\min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

such that:

$$\sum_{j=1}^{n} x_{ij} = s_i \qquad \text{and} \qquad \sum_{i=1}^{m} x_{ij} = d_j$$

All x_{ij} should be non-negative.

How do we ensure this is balanced?

$$\text{Total Supply} = \text{Total Demand} \qquad \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

What does that look like in the array? (The sum of the first m rows is the total supply, the sum of the last n rows is the demand).

Example

PowerCo has three electric power plants (1, 2, 3) that supply the needs of four cities (A, B, C, D). See Table 1 for the amount of power available to each city. See Table 1 for the power demands in these cities which occur at the same time (2PM). The costs of sending a million kilowatt hours from the plant to the city depends on several factors, and the cost table is shown as well. Formulate an LP to minimize the cost of meeting each city's peak demands.

	A	*B*	*C*	*D*	Supply
1	8	6	10	9	35
2	9	12	13	7	50
3	14	9	16	5	40
Demand	45	20	30	30	

Now, the goal of the transportation problem is to "fill in the boxes". Try one. Here's an example (be sure your column and row sums match up!)

Here is a possible distribution scheme. Calculate its cost:

	City A	City B	City C	City D	Supply
	8	6	10	9	
Plant 1		10	25		35
	9	12	13	7	
Plant 2	45		5		50
	14	9	16	5	
Plant 3		10		30	40
Demand	45	20	30	30	125

This cost is $1150.00. We're going to see how to do these assignments in the next section, but in this section we look at having Excel compute a solution. In this case, we'll treat the problem as any other linear program. Our class website has the example for the power distribution problem.

Key Point: Notice how we organized the information from a typical transportation problem!

Handout for Class

Try to find the cheapest way to distribute the power:

	City A	City B	City C	City D	Supply
Plant 1	8	6	10	9	35
Plant 2	9	12	13	7	50
Plant 3	14	9	16	5	40
Demand	45	20	30	30	125

Setting up and Solving in Excel

Here is a good way to organize the power distribution problem. I've used one array of constants for the costs, and reserved space for the array of variables (x_{ij}). Then we keep track of the supplies down the last column (sums), and the demands across the bottom row. The total cost is then a "dot product" between the cost array and variable array (as shown in the formula boxes). (NOTE: To type an equal sign and not have a formula, type a blank space first). You can download this from our class website as a template.

	A	B	C	D	E	F	G	H	I	J
1	**Power Distribution Problem (Example from Text, 7.1)**									
2										
3		**Unit Cost**								
4				**City A**	**City B**	**City C**	**City D**			
5			**Plant 1**	8	6	10	9			
6			**Plant 2**	9	12	13	7			
7			**Plant 3**	14	9	16	5			
8										
9										
10		**Distribution**								
11		**(KwH)**		**City A**	**City B**	**City C**	**City D**	Total Sent		Supply
12			**Plant 1**					=SUM(D12:G12)	=	35
13			**Plant 2**					=SUM(D13:G13)	=	50
14			**Plant 3**					=SUM(D14:G14)	=	40
15			Total Received	=SUM(D12:D14)	=SUM(E12:E14)	=SUM(F12:F14)	=SUM(G12:G14)			
16				=	=	=	=			Total Cost:
17			Demand	45	20	30	30			=SUMPRODUCT(D5:G7,D12:G14)

To set up the solver:

- We want to **minimize** cell `J17`
- The conditions are that the column sums (in cells D15 to G15) are equal to the demand (in cells D17 to G17), and the row sums (in cells H12-H14) are equal to the supply (in cells J12-14). All told, there are then 4+3=7 equality constraints.

The solver should give you the following result:

	A	B	C	D	E	F	G	H	I	J
1	**Power Distribution Problem (Example from Text, 7.1)**									
2										
3		**Unit Cost**								
4				City A	City B	City C	City D			
5			Plant 1	8	6	10	9			
6			Plant 2	9	12	13	7			
7			Plant 3	14	9	16	5			
8										
9										
10		**Distribution**								
11		**(KwH)**		City A	City B	City C	City D	Total Sent		Supply
12			Plant 1	0	10	25	0	35	=	35
13			Plant 2	45	0	5	0	50	=	50
14			Plant 3	0	10	0	30	40	=	40
15			Total Receive	45	20	30	30			
16				=	=	=	=			Total Cost:
17			Demand	45	20	30	30			1020

In Class Exercise

(7.1, #1) A company supplies goods from two warehouses to three customers (who still want 30 units each). Unfortunately, warehouse 1 and 40 units and warehouse 2 has 30 units so we cannot fulfill all the orders. The costs of shipping are the same as before. There will be a penalty for each unit of unmet customer demand: $90, $80, and $110 for customers 1, 2 and 3 respectively. Formulate a balanced transportation problem to minimize the shortage and shipping costs.

SOLUTION: Have a *virtual warehouse* that will ship to the customers. That makes the tableau:

	Cust 1	Cust 2	Cust 3	Supply
Wh 1	15	35	25	40
Wh 2	10	50	40	30
Dummy	90	80	110	20
Demand	30	30	30	90

Part 2

Suppose that Warehouses 1 and 2 each have 50 units. Formulate a balanced transportation problem for this new situation.

	Cust 1	Cust 2	Cust 3	Virtual Cust	Supply
Wh 1	15	35	25	0	50
Wh 2	10	50	40	0	50
Demand	30	30	30	10	90

Added HW for 7.1

7.1.13 One of the main products of the P & T Company is canned peas. The peas are prepared at three canneries (near Bellingham, WA; Eugene, OR; and Albert Lea, MN), and then shipped by truck to four distribution warehouses in the western US (Sacramento, Salt Lake City, Rapid City, and Albuquerque).

Because the shipping costs are a major expense, management is initiating a study to reduce them as much as possible.

For the upcoming season, an estimate has been made of the output from each cannery, and each warehouse has been allocated a certain amount from the total supply of peas.

This information (in units of truckloads), along with the transportation costs, is given in the table below. There are 300 truckloads to be distributed. Use a spreadsheet to formulate this as a transportation problem, and solve it using the spreadsheet's solver. You may use the spreadsheet on the class website as a template.

Shipping Costs

	Warehouse				
	1	2	3	4	Output
Cannery 1	464	513	654	867	75
Cannery 2	352	416	690	791	125
Cannery 3	995	682	388	685	100
Allocation	80	65	70	85	

7.1.14 In a balanced transportation problem, show that any supply constraint exactly equals the sum of the demand constraints, minus the sum of the other supply constraints. This shows that one row is a linear combination of the others.