Group Work: Duality and Sensitivity Analysis

In the textbook's "Dakota Problem", we are making desks, tables and chairs, and we want to maximize profit given constraints on lumber, finishing and carpentry (resp).

For the primal, let x_1, x_2, x_3 be the number of desks, tables and chairs we make (resp), where the original (max) tableau is as given below:

x_1	x_2	x_3	s_1	s_2	s_3	rhs
-60	-30	-20	0	0	0	0
8	6	1	1	0	0	48
4	2	$\frac{3}{2}$	0	1	0	20
2	$\frac{3}{2}$	$\frac{1}{2}$	0	0	1	8

$\rightarrow$

x_1	x_2	x_3	s_1	s_2	s_3	rhs
0	5	0	0	10	10	280
0	-2	0	1	2	-8	24
0	-2	1	0	2	-4	8
1	$\frac{5}{4}$	0	0	$-\frac{1}{2}$	$\frac{3}{2}$	2

1. Write the down vectors/matrices that we typically use in our computations. Namely, $\mathbf{c}, \mathbf{c}_B, B$, and B^{-1}.

2. Using our vector notation, if $\mathcal{B}$ gives the optimal basis, how do we compute the dual, $\mathbf{y} =$?

3. Write down the dual (either as an initial tableau or in "normal form").

4. Using the optimal Row 0 from the primal, write down the solution to the dual:

5. In our "normal form", we have $A\mathbf{x} \le \mathbf{b}$ for the primal and $A^T\mathbf{y} \ge \mathbf{c}$ for the dual. We will define two "slacks"[1]

   - The "slack" for the primal, given $\mathbf{x}$: $\mathbf{b} - A\mathbf{x}$. Compute the current slack for the primal.

   - The "slack" for the dual, given $\mathbf{y}$: $A^T\mathbf{y} - \mathbf{c}$. Compute the current slack for the dual. You can use your answer to (3) if necessary.

6. What is the shadow price for each constraint?

7. Write down the inequalit(ies) we need for Δ if we change the coefficient of x_2 from 30 to $30 + \Delta$, and we want the current basis to remain optimal.

8. Write down the inequalit(ies) we need for Δ if we change the coefficient of x_3 from 20 to $20 + \Delta$, and we want the current basis to remain optimal.

9. How does changing a *column* of A effect the dual? Use this to see what would happen if we change the column for x_2 (tables) to be $[5, 2, 2]^T$- Is it now worth it to make tables?

10. How does creating a new column of A effect the dual? Use this to see if it makes sense to manufacture footstools, where we sell them for $15 each, and the resources are $[1, 1, 1]^T$.

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[1]Sorry, the vocabulary is related to the "slack variable", but we're using "slack" in a different context now. Ask if you're ever not sure which we're talking about.