Review Material, Exam 1, Ops Research

The exam will cover material from Chapters 3 and 4. Excluded sections: 3.6, 3.7, 4.9, 4.10, 4.15. You might note that 4.14 is about unrestricted variables, which we covered early, and 4.17 is about the Excel solver (I won't ask about the software in-class).

Background Material: Linear Algebra

Though these may not be asked explicitly, you should be able to do the following (and may be part of solutions):

1. Be able to solve $A\mathbf{x} = \mathbf{b}$ for different types of matrices (if A is square and invertible, and if it is tall and full rank, tall and not full rank, wide and full rank, wide and not full rank).
2. Find a basis for the null space of A; solve $A\mathbf{x} = \mathbf{0}$ and put your answer in vector form.
3. Be able to compute the determinant and inverse of a matrix A
4. Note that our book uses "ERO" for elementary row operation.
5. Define what it means for a set of vectors to be linearly independent. Given $A\mathbf{x} = \mathbf{b}$, be able to find a basic feasible solution.
6. What is a "bounded set" in $\mathbb{R}^n$? What is a "closed" set in $\mathbb{R}^n$?

Definitions:

- Ch 3:

  linear function, linear inequalities, objective function, optimal solution, feasible set, linear program, isoprofit/isocost lines, binding (non-binding) constraints, convext set, extreme point, basic and non-basic variables, BFS, slack variables, excess variables, standard form for an LP.

- Ch 4:

  Direction of unboundedness, convex combination, adjacent BFS, degenerate BFS(*), artificial variables, URS variables, stalling, cycling.

(*) Book uses "degenerate LP" which is ambiguous, so I won't ask about that. Remember that there are two different types of unboundedness- An unbounded LP, an unbounded feasible set.

Theorems:

- **Theorem 1** says that extreme points are the same BFS (and are the same as corner points). Page 132, Section 4.2.

- **Direction of Unboundedness Theorem** is actually Problem 6, Section 4.4. This gives an important characterization of the direction of unboundedness: "For an LP in standard form with constraints $A\mathbf{x} = \mathbf{b}$, and $\mathbf{x} \geq 0$, then $\mathbf{d}$ is a direction of unboundedness if and only if $A\mathbf{d} = 0$ and $\mathbf{d} \geq 0$."

- **Theorem 2: The Representation Theorem** Let our LP be in standard form with constraints $A\mathbf{x} = \mathbf{b}$. Let $\mathbf{b}_i, i = 1, 2, \ldots, n$ be the corner points of the feasible set with possible direction of unboundedness $\mathbf{d}$. Then any feasible point $\mathbf{x}$ may be written as the convex combination of the corner points and the direction of unboundedness:

  $$\mathbf{x} = c\mathbf{d} + \sum_{i=1}^{N} \sigma_i \mathbf{b}_i$$

  where $\mathbf{x}$ is any feasible point, $c = 0$ or $c = 1$ (depends on whether we have a direction of unboundedness), and $\sigma_i \geq 0$ with $\sum_{i=1}^{n} \sigma_i = 1$. This is on page 135, Section 4.3.

  Key Skill: Be able to actually compute this sum given a specific convex set and

- **Theorem 3: The Fundamental Theorem of Linear Programming**

  If an LP has an optimal solution, then it has an optimal BFS (or corner point, or extreme point).

Key Algorithms

- The (basic) Simplex Algorithm.
- Big-M
- Two Phase
- Goal Programming

Skills

1. Be able to discuss the four assumptions for a linear program: Proportionality, Additivity, Divisibility, and Certainty. NOTE: I won't make you memorize the list of things, but do be prepared to discuss what each means.

2. Translate unrestricted variables so that all variables are non-negative.

3. Be able to show that a given set is convex, use convexity in other arguments (examples are in the review questions).

4. Be able to set up and solve an LP graphically. Given the objective function, be able to state the direction in which the function increases (or decreases) the fastest.

   Know the geometry for each of the three possible outcomes (unique solution, infinite solutions, no solution).

5. Be able to set up an LP generally, and of specific types: A diet problem, a work scheduling problem, a production process model, blending, and multiperiod problems (like the sailboat example).

6. Be able to translate a LP into standard form given "less than or equal to" constraints, "greater than or equal to" constraints, and change the variables so that they are all non-negative.

7. Given an LP in standard form with an unbounded set, be able to determine a direction of unboundedness.

8. Given a convex set (bounded or unbounded) and a point in the convex hull, be able to compute the coefficients from the Representation Theorem (Theorem 2).

9. Be able to write the simplex tableau from the linear program (for both a maximization and a minimization problem).

10. Be able to solve a linear program (a maximization problem with "less than or equal to" constraints) using the Simplex Method.

11. Be able to solve a linear program (mixed constraints) using the Big-M method, and be able to interpret its final tableau. Same for the Two-Phase Method.

12. Given a tableau, be able to tell if it is a terminal tableau, and interpret what the solution is (unique, multiple, unbounded, infeasible).

13. Set up the tableau for the Goal Programming (4.16) method, and be able to interpret its "final" tableau.

14. (For something that might be take home) Be able to use either LINDO, LINGO or a spreadsheet program to solve an LP.