Homework Addendum for 4.1-4.3

1. Show that the intersection of two convex sets is convex.

2. If we define a *hyperplane* as the set of vectors $\mathbf{x}$ so that $\mathbf{c}^T\mathbf{x} - b = 0$ for some constant vector $\mathbf{c}$ and scalar b, then show that a hyperplane is convex.

3. Show that the convex hull of a given set of k vectors is convex.

4. Find a **convex combination** of the vertices of the polygonal region below for the point $(1, 2)$. You may do this all in two dimensions.

$$\begin{aligned} x_1 + \quad & x_2 \leq 5 \\ 3x_1 + \quad & 2x_2 \leq 12 \\ x_1, \quad & x_2 \geq 0 \end{aligned}$$

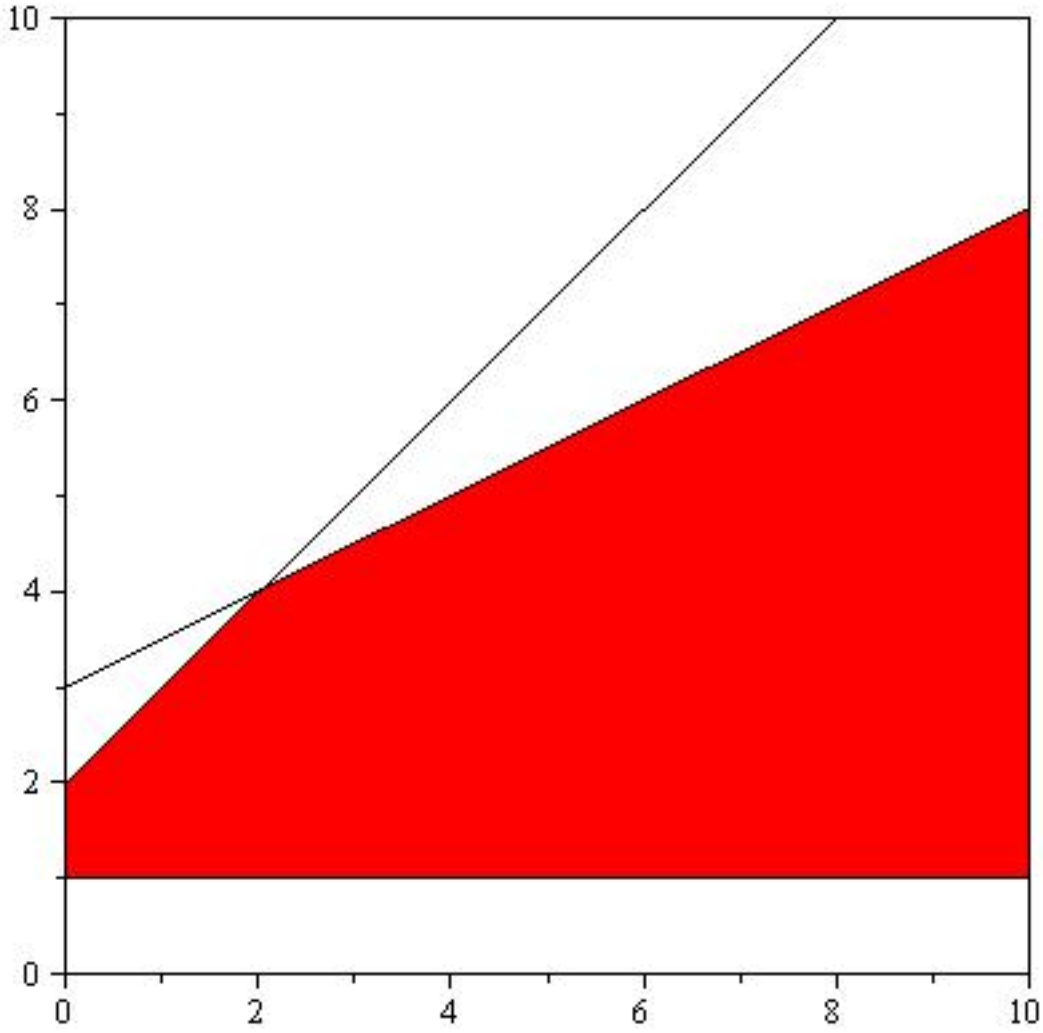
5. Consider the feasible set:

$$\begin{aligned} 2x + \quad & y \geq 4 \\ x + \quad & 2y \geq 6 \\ -x \quad & +2y \geq 2 \\ x, \quad & y \geq 0 \end{aligned}$$

   (a) Write the feasible set in standard form.

   (b) Find a direction of unboundedness, $\mathbf{d}$.

   (c) Write the point corresponding to $(2, 4)$ using Theorem 2, page 135. Be sure to pay attention to the appropriate dimensions!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