Homework, Section 6.10 (Complementary Slackness)

1. Consider the LP:

$$\begin{aligned} \max z = \quad & 2x_1 + 5x_2 + 3x_3 \\ \text{st} \quad & 2x_1 + x_2 + 2x_3 \leq 10 \\ & \tfrac{3}{2}x_1 + 6x_2 - 2x_3 \leq 18 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

   (a) State the dual.

   (b) Given that $x_1 = 0, x_2 = 3, x_3 = 0$ is an basic solution to the LP, use complementary slackness to find the complementary basic solution to the dual.

   (c) Given that $x_1 = 4, x_2 = 2, x_3 = 0$ is an basic solution to the LP, use complementary slackness to find the complementary basic solution to the dual. Are these solutions optimal?

   (d) Given that $x_1 = 0, x_2 = 4, x_3 = 3$ is an basic solution to the LP, use complementary slackness to find the complementary basic solution to the dual. Are these solutions optimal?

2. Consider the LP:

$$\begin{aligned} \max z = \quad & 5x_1 + 10x_2 \\ \text{st} \quad & x_1 + 3x_2 \leq 50 \\ & 4x_1 + 2x_2 \leq 60 \\ & x_1 \leq 5 \\ & x_1, x_2 \geq 0 \end{aligned}$$

   (a) State the dual.

   (b) Given that $x_1 = 5, x_2 = 15$ is an optimal solution to the LP, use complementary slackness to find the optimal solution to the dual.