# Basic Problem (max)

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $rhs$ |
|---|---|---|---|---|---|
| $-3$ | $-5$ | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 4 |
| 0 | 2 | 0 | 1 | 0 | 12 |
| 3 | 2 | 0 | 0 | 1 | 18 |

Write the dual:

$$\begin{array}{llllll} \min & w & = 4y_1 & +12y_2 & +18y_3 & \\ & \text{st} & y_1 & & +3y_3 & \geq 3 \\ & & & y_2 & +2y_3 & \geq 5 \end{array}$$

For future reference:

$$\Rightarrow \quad e_1 = -3 + y_1 + 3y_3$$

$$e_2 = -5 + y_2 + 2y_3$$

# Basic Problem (max)

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $rhs$ |
|---|---|---|---|---|---|
| $-3$ | $-5$ | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 4 |
| 0 | 2 | 0 | 1 | 0 | 12 |
| 3 | 2 | 0 | 0 | 1 | 18 |

- Current basis is: $\mathcal{B} = \{s_1, s_2, s_3\}$
- $\mathbf{c}_B^T = [0, 0, 0]$
- Dual: $\mathbf{y}^T = \mathbf{c}_B^T B^{-1} = [0, 0, 0]$
- And the excess variables:

$$\begin{aligned} e_1 &= -3 + y_1 + 3y_3 = -3 \\ e_2 &= -5 + y_2 + 2y_3 = -5 \end{aligned}$$

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $rhs$ |
|---|---|---|---|---|---|
| $-3$ | $-5$ | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 4 |
| 0 | 2 | 0 | 1 | 0 | 12 |
| 3 | 2 | 0 | 0 | 1 | 18 |

This means that we have the following

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| 0 | 0 | 4 | 12 | 18 |

| $e_1$ | $e_2$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|
| $-3$ | $-5$ | 0 | 0 | 0 |

The solution to the dual is FEASIBLE, the solution to the PRIMAL is INFEASIBLE.

Note where the zeros appear in each solution...

Change basis $\mathcal{B} = \{s_1, s_2, x_1\}$, and recompute:

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $rhs$ |
|---|---|---|---|---|---|
| 0 | $-3$ | 0 | 0 | 1 | 18 |
| 0 | $-2/3$ | 1 | 0 | $-1/3$ | $-2$ |
| 0 | 2 | 0 | 1 | 0 | 12 |
| 1 | $2/3$ | 0 | 0 | $1/3$ | 6 |

$$\mathbf{y} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Solve the primal and the dual:

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| 6 | 0 | $-2$ | 12 | 0 |

| $e_1$ | $e_2$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|
| 0 | $-3$ | 0 | 0 | 1 |

Feasibility?
The primal $\mathbf{x}$ is not feasible, the dual $\mathbf{y}$ is not feasible.

New basis: $\mathcal{B} = \{x_1, x_2, s_3\}$.

| $x_1$ | $x_2$ | $s_2$ | $s_2$ | $s_3$ | *rhs* |
|---|---|---|---|---|---|
| 0 | 0 | 3 | 5/2 | 0 | 42 |
| 1 | 0 | 1 | 0 | 0 | 4 |
| 0 | 1 | 0 | 1/2 | 0 | 6 |
| 0 | 0 | −3 | −1 | 1 | −6 |

$$\mathbf{y} = \begin{bmatrix} 3 \\ 5/2 \\ 0 \end{bmatrix}$$

The solutions to primal and dual are:

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| 4 | 6 | 0 | 0 | −6 |

| $e_1$ | $e_2$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|
| 0 | 0 | 3 | 5/2 | 0 |

Feasibility?

This is dual feasible, but primal infeasible.

Final basis: $\mathcal{B} = \{x_1, x_2, s_1\}$

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $rhs$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | $3/2$ | 1 | 36 |
| 1 | 0 | 0 | $-1/3$ | $1/3$ | 2 |
| 0 | 1 | 0 | $1/2$ | 0 | 6 |
| 0 | 0 | 1 | $1/3$ | $-1/3$ | 2 |

$$\mathbf{y} = \begin{bmatrix} 0 \\ 3/2 \\ 1 \end{bmatrix}$$

Solutions to the primal, dual are:

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| 2 | 6 | 2 | 0 | 0 |

| $e_1$ | $e_2$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $3/2$ | 1 |

Feasibility?
Both the primal and dual are feasible (so optimal).

# Is there an interesting pattern in the solutions?

The solutions to primal and dual are:

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| | | | | |

| $e_1$ | $e_2$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|
| | | | | |

The product $e_j x_j = 0$ and $s_j y_j = 0$.
Does this makes sense?

- If $s_j > 0$ (slack in constraint $j$), does it matter if we increase the right hand side of constraint $j$?

  No- The shadow price must be zero.
- We have a similar argument for the dual...

Next: Section 6.10, Complementary Slackness