Example: BFS and Vertices

Original example:

$$\begin{array}{rlll} \min_x & 3x_1- & 4x_2 & \\ \text{s.t.} & -2x_1+ & x_2 & \leq 1 \\ & -x_1+ & 2x_2 & \geq 5 \\ & x_1+ & x_2 & \leq 7 \end{array}$$

with $x_1, x_2, x_3 \geq 0$.

This was translated into standard form:

$$\begin{array}{rrrrrrr} \min_x & 3x_1- & 4x_2 & +0s_1 & +0e_2 & +0s_3 & \\ \text{s.t.} & -2x_1 & +x_2 & +s_1 & & & = 1 \\ & -x_1 & +2x_2 & & -e_2 & & = 5 \\ & x_1 & +x_2 & & & +s_3 & = 7 \end{array} \quad \Rightarrow \quad \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ e_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$$

A summary of all the basic solutions:

Basic Variables	Solution	Feasible?	Objective
$1, 2, 3$	$3, 4, 3$	Yes	-7
$1, 2, 4$	$2, 5, 3$	Yes	-14
$1, 2, 5$	$1, 3, 3$	Yes	-9
$1, 3, 4$	$7, 15, -12$	No	
$1, 3, 5$	$-5, -9, 12$	No	
$1, 4, 5$	$-0.5, -4.5, 7.5$	No	
$2, 3, 4$	$7, -6, 9$	No	
$2, 3, 5$	$2.5, -1.5, 4.5$	No	
$2, 4, 5$	$1, -3, 6$	No	
$3, 4, 5$	$1, -5, 7$	No	

The original feasible set (Can you tell which BFS corresponds to which vertex?)